$C_2 = -(k+1)\cdot\mu\cdot t_0$. Также как и это было сделано выше, нелинейное решение сращивается с линейным и имеет окончательный вид:

$$M_1 - \mu\cdot t = \frac{2\cdot\left(x - a_0\cdot t - \dfrac{a_0\cdot\mu\cdot t^2\cdot\varepsilon}{2} + a_0\cdot(k+1)\cdot t_2\right)}{a_0\cdot(\kappa+1)\cdot\varepsilon\cdot\left(t + \dfrac{2\cdot t_2}{(\kappa+1)\cdot\varepsilon}\right)} \quad (3.31).$$

Для того случая когда $\mu < 0$ все предыдущие вычисления формально имеют тот же вид. В этом сценарии силы сопротивления превосходят силу ветра и анализ предыдущих соотношений позволяет сделать вывод что высота волны $h - h_0$ увеличивается с ростом времени, а скорости частиц M_1 уменьшаются.

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MODELING SPREAD OF LIGHT IN LAYERED-HETEROGENEOUS MEDIUM

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ABSTRACT

The article considers the possibility of modeling the propagation of light in a layered inhomogeneous medium. The knowledge of students in the field of mathematics and informatics makes it possible to organize a new kind of educational activity, like mathematical and computer modeling, while studying the phenomenon of light propagation in a layered heterogeneous medium.

АННОТАЦИЯ

В статье рассматривается возможность моделирования распространения света в слоисто-неоднородной среде. Знание студентов в области математики и информатики позволяет при изучении явления распространения света в слоисто-неоднородной среде организовать новый вид учебной деятельности, как математическое и компьютерное моделирование.

Keywords: optics, layered-heterogeneous medium, Snell's law, mathematical and computer modeling.

Ключевые слова: оптика, слоисто-неоднородная среда, закон Снеллиуса, математическое и компьютерное моделирование.

Optics of heterogeneous medium is quite extensive and completely not simple area of physics, which has a great scientific-practical importance. For electromagnetic waves of a certain frequency, the earth's atmosphere introduces layered-heterogeneous medium, the refractive index of which continuously decreases with altitude. In such environment the electromagnetic wave spreads curvilinearly, which cannot be demonstrated in laboratory conditions. Thus, familiarizing students with the physical ideas of optics of heterogeneous medium, with which they do not meet in the main course, represents big cognitive interest.

In heterogeneous medium, the idea of the propagation of light along the rays is preserved, and the geometric shape of the ray can be uniquely determined from Snell's law with limit transition way [5, c. 31].

As an example, consider the simplest case, when the refractive index of medium changes in only one direction and depends on one coordinate. Such propagation of light is called layered-heterogeneous medium [3, c. 84].

Imagine an optically heterogeneous medium, the refractive index n of which is a function of only one coordinate y:

$$n = n(y). \quad (1)$$

Such a medium, as already noted, is called layered-heterogeneous. As mentioned above, earth's atmosphere is an example of it. Although the dependence of refractive index *n* of medium on the coordinate *y* (1) is difficult, medium in the first approach for a bounded region can always be assumed to be linear:

$$n = n_0+ky \tag{2}$$

Where, n_0 – refractive index medium in points with coordinate $y = 0$, k = $^{dn}/_{dy}$ - constant gradient of refraction.

By choosing from an diverging ray from a point source of light an arbitrary ray, going under angle $\varphi_1 < \pi/2$ to axis *y* (Fig.1), Find the trajectory of the selected ray. The optical heterogeneous medium is split into plane-parallel layers, the perpendicular axes of *y* are so thin, that inside each of the layers the light moves rectilinearly, and on the boundary between neighboring layers is refracted so, that the trajectory of the light ray is a broken line. According to the law of refraction

$$\frac{sin\varphi_1}{sin\varphi_2} = \frac{n_2}{n_1}, \frac{sin\varphi_2}{sin\varphi_3} = \frac{n_3}{n_2}, \frac{sin\varphi_3}{sin\varphi_4} = \frac{n_4}{n_3}, , , ,$$

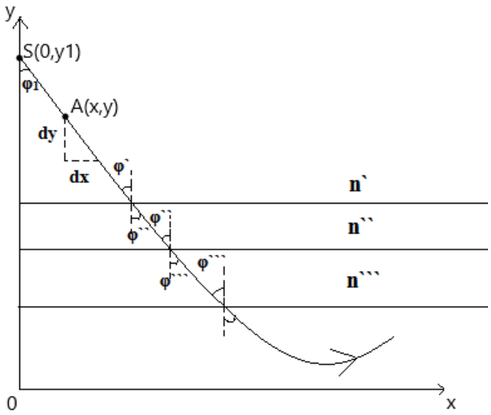

Figure 1. An arbitrary beam, a divergent beam from a point source of light

or $n_1 sin\varphi_1 = n_2 sin\varphi_2 = n_3 sin\varphi_3 = , , ,$ that is $n(y)\ sin\,\varphi = const.$

Then in limit instead of broken, we will get curvilinear trajectory light ray. It is obvious that in this case:

$$n(y)\ sin\,\varphi = const \text{ or } n(y)\ sin\,\varphi = m \tag{3}$$

where, *m* –some constant. The physical meaning of a constant *m* is quite simple: this is the meaning of the refractive index in that plane of the medium, at the points which $sin\,\varphi = 1$, that is there, where the light ray is directed perpendicular to the axis *y*.

For an arbitrary point of a trajectory, we can write

$$tq\,\varphi = -\ \frac{dn}{dy} \tag{4}$$

Since $tq\,\varphi = \dfrac{\sin\varphi}{\sqrt{1-\sin^2\varphi}}$, from (4) and (3) following, that

$$\frac{dx}{dy} = -\frac{m}{\sqrt{n^2(y)-m^2}}. \tag{5}$$

Integration of the equation (5) gives

$$x = \int_0^x dx = -m\int_{y_1}^{y_2}\frac{dy}{\sqrt{n^2(y)-m^2}}. \tag{6}$$

In equation (6) we will change the variable, by crossing *y* to *n:*

$x = -m\int_{y_1}^{y_2} \frac{dy}{\sqrt{n^2(y) - m^2}} = -\frac{m}{k}\int_{n_1}^{n_2} \frac{dn}{\sqrt{n^2(y) - m}^2}$,where n_1 – refractive index of the medium at points with coordinates $y_1 : n_1 = n_0 + ky_1$.

If we use substitution $n = mchz$, then it is not difficult to make sure, that the antiderivative of the last integral is equal to the hyperbolic arc cosine. Therefore

$x = -\frac{m}{k} arch\frac{n}{m} \mid_{n_1}^{n} = -\frac{m}{n}(arch\frac{n}{m} - arch\frac{n_1}{n})$. From here $arch\frac{n}{m} = -\frac{k}{m}x + arch\frac{n_1}{m}$.

Taking both sides of this formula a hyperbolic cosine, we will get

$$n = mch(-\frac{k}{m}x + arch\frac{n_1}{m}) \qquad (7)$$

Considering (2) and paying attention to parity of the hyperbolic cosine, rewrite (7) in the form of

$$y = -\frac{n_0}{k} + \frac{m}{k}ch\left[\ \frac{k}{m}(x - arch\frac{n_1}{m})\ \right] \qquad (8)$$

The equation (8) in an explicit form describes the trajectory of the propagation of light in a layered heterogeneous medium with a constant refractive index gradient.

Function (8) is a hyperbolic cosine. To approximately build the graph, we need to find the coordinates of the minimum of this function. Differentiating the function (8) by coordinate x, we will get

$$\frac{dy}{dx} = sh\ (\frac{k}{m}\ x\ - arch\frac{n_1}{m}). \qquad (9)$$

Equating this derivative to zero, we find that this equality will be satisfied, if $\frac{k}{m}\ x\ - arch\frac{n_1}{m} = 0$.

From here the abscissa of the minimum

$$x_M = \frac{k}{m}\ x\ - arch\frac{n_1}{m} \qquad (10)$$

Substitution this abscissa value in equation (8), we will find the coordinate of the same point M:

$$y_M = \frac{-n_0 + m}{k}\ . \qquad (11)$$

The hyperbolic cosine is symmetric with relatively to the vertical axis, passing through the minimum point M. An approach the graph of the function constructed in the Cartesian coordinate system (8) is shown in Fig . 2

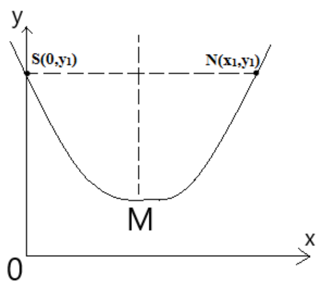

Figure 2. The graph of the function constructed in the Cartesian coordinate system

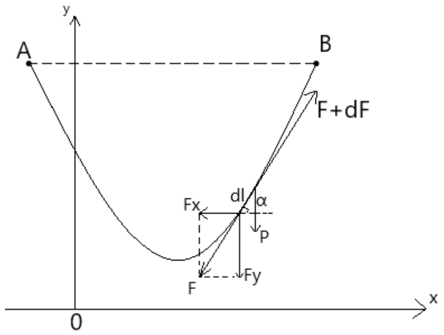

Figure 3.Homogeneous inextensible and heavy chain, the ends of which are fixed at points A and B

To demonstrate the propagation of light in a medium with a constant refractive index gradient, we model the path of the light ray by a chain line, since modeling allows us to replace the object under study with another, specially created for this, but preserving the characteristics of the real object, necessary for its study [2, c.137]. To do this, take a break from the optics and remember the statistics. Imagine a flexible homogeneous inextensible and heavy chain, the ends of which are fixed at points A and B (Fig.3). If the length of the thread is greater than the AB, the chain will hang. Let's find an equation that describes the position of the sagging chain.

For this we denote the mass of a unit of the chain length by ρ. Gravity acting on the element of length dl of the thread

$$P = \rho g\ dl, \tag{9}$$

Where g ─ is the acceleration of gravity. Since the chain is located in equilibrium, the sum of the forces acting on any of its elements dl, is equal to zero (Fig.): $\boldsymbol{P} + \boldsymbol{F} + (\boldsymbol{F} + d\boldsymbol{F}) = 0$, where $\boldsymbol{F}$ and $\boldsymbol{F} + d\boldsymbol{F}$ ─ are the forces of chain tension. Passing from the vector equation to the equations in the projections, we obtain

$- F_x + (F_x + dF_x) = 0,$

$- F_y + (F_y + dF_y) - P = 0.$

From here $dF_x = 0$ (that is $F_x = \text{const}$) and $dF_y = P$. It can be seen in the figure that $dy/dx = tg\alpha = F_y / F_x$. Differentiating this formula by x and given that $F_x = \text{const}$, a $dF_y = \rho g\ dl$, we obtain

$$\frac{d^2y}{dx^2} = \frac{1}{Fx}\frac{dFy}{dx} = \frac{\rho g}{Fx}\frac{dl}{dx}. \tag{10}$$

Since $dl = \sqrt{dx^2 + dy^2}$ or $\frac{dl}{dx} = \sqrt{1 + (dy/dx)^2}$, then denoting $F_x / \rho g = a$, we obtain the differential equation

$$a\frac{d^2y}{dx^2} = \sqrt{1 + (dy/dx)^2}. \tag{11}$$

The equation can be integrated by using the substitution $dy/dx = sh\ z$. Then $d^2y/dx^2 = sh\ z \cdot dz/dx$.

Pay attention to that $1 + sh^2 x = ch^2 x$, equation () we reduce it to the form $a \cdot dz/dx = 1$. From here $z = \frac{1}{a}x + C_1$, $dy/dx = sh\ (\frac{1}{a}x + C_1)$. Therefore, we finally obtain

$$y = a\ sh\ (\frac{1}{a}x + C_1) + C_2. \tag{12}$$

To determine the constant integrations C_1 and C_2 , it is necessary to use some initial conditions.

Comparison of formulas (8) and (12) shows that the curve along which the chain sags is described by the same equation as the light propagation path in a layered-heterogeneous medium with a constant refractive index gradient, since both formulas are the equation of a hyperbolic function. This allows us to use the saggy chain as a physical model of the trajectory of the light ray (Fig.4). Simple experiments shows that the thread must be heavy, homogeneous, flexible and inextensible, so it is best to use a chain to demonstrate the trajectory of light in heterogeneous medium. Therefore, the graph of the hyperbolic is called chain line.

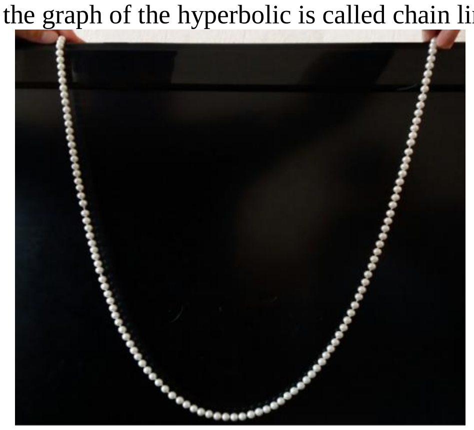

Figure 4. The physical model of the trajectory of the light ray

The initial «advancement» of students in the field of informatics allows organizing a new kind of educational activity, like computer modeling when studying the phenomenon of light propagation in a layered heterogeneous medium. Computer modeling, carrying out of computing experiment is one of modern methods of research of the physical phenomena.

The construction of graphs in the MathCAD system is an easy task, as the MathCAD package has a powerful mathematical device that allows performing symbolic calculations, solving algebraic and differential equations systems, operations with vectors and matrices, writing programs, drawing graphics and surfaces, etc.[4, c. 4].

Use the *Graph* toolbar to construct the light propagation trajectory, i.e., the graph of dependence of *n* on *y* [1, c. 335].

Demonstration of the trajectory of a light ray with the help of a sagging chain and carrying out research in the form of a computer experiment makes it possible to obtain knowledge, skills (and not only in the field of physics) that contribute to the generation of motivation for training activities that is so necessary in modern conditions and the formation of a whole range of competencies for future specialists.

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