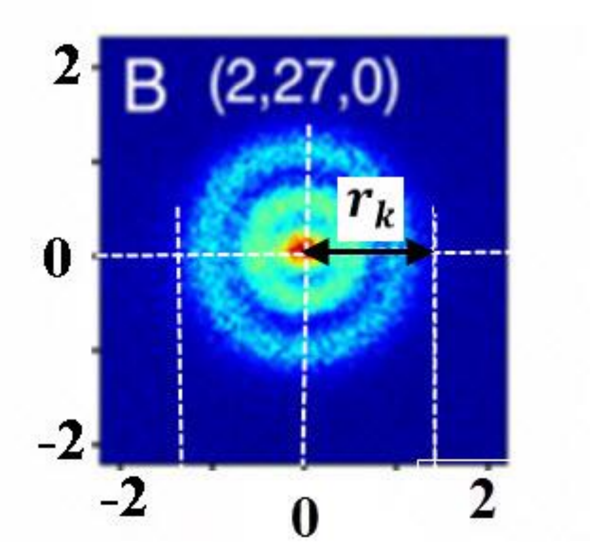

Рис 5. Возбуждённое состояние атома водорода и внешний радиус кольцеобразной структуры.

На основе вышеизложенного можно прийти к выводу, что для получения достоверных научных результатов при исследовании атомных структур, необходимо обратиться к законам фундаментальной механики и электродинамики.

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METHODOLOGICAL ERRORS ALLOWED IN THE CALCULATION OF ELECTROMAGNETIC FIELDS USING THE MAXWELL INTEGRAL EQUATIONS

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ABSTRACT

On the example of solving two problems, methodological errors are considered that are present in solving these and similar problems in basic university textbooks in the course of general physics. It is shown that in the case of solving the problem of finding the magnetic field of an infinite solenoid with current outside the solenoid

and a toroid with current outside the toroid, these errors lead to an incorrect result, which has been rewritten from textbook to textbook for more than 40 years.

Keywords: symmetry considerations, magnetic field of the solenoid, Maxwell integral equations

Introduction

Many serious mathematicians do not like to read scientific treatises if there are few formulas because of the "ambiguity of words."

The brilliant experimenter Michael Faraday received only primary education in childhood. Therefore, he described his many experiments on electricity and magnetism in two very thick volumes of The Treatise on Electricity in excellent english, trying to write so that what he wrote was correctly and equally understood by any interested reader. For this he needed a lot of words.

The young mathematician James Clark Maxwell carefully read the work of Faraday and set himself the task of "translating all the basic laws established and described by Faraday from English into the language of mathematics." As a result, he received four short equations, which immortalized his name, which together with the "Lorentz force" solve in principle any problem of classical electrodynamics and optics.

Applying these equations for a homogeneous isotropic medium with constant values of electromagnetic constants, Maxwell showed that light is an electromagnetic wave and derived the famous "Maxwell formula" connecting these constants with the speed of light in a given medium.

Already much later, after the creation of the special theory of relativity by A. Poincare and A. Einstein, it turned out that the Maxwell equations are invariant with respect to Lorentz transformations, i.e. have the same form in any inertial system moving relative to the phenomena under consideration at any speed, up to the speed of light in vacuum. What neither Faraday nor Maxwell had any idea.

"When you solve problems with the help of these equations, it creates the feeling that they are much smarter than us" ...

Therefore, in the course of general physics, increased attention is paid to the good assimilation of the lecture material related to the Maxwell equations.

Unfortunately, over 40 years of experience in teaching a course in general physics convinced the author that the technique published in the recommended textbooks for solving approximate problems of finding the strength of the electrostatic field due to existing methodological errors in the text cannot be understood with independent reading even by strong students.

This is manifested in the fact that, trying to honestly learn what they read from the textbook, strong students, and sometimes teachers, can not even begin to solve other similar problems without the active help of qualified professors, if they are still nearby.

Moreover, the lack of a common methodologically correct and understandable approach leads to an incorrect solution to the problem of finding the magnetic field of an infinite solenoid outside the solenoid, which has been translated from textbook to textbook, both in our country and abroad, for more than 40 years.

Purpose of the study

Below, on examples of solving two problems, methodologically correct from the author's point of view, errors and inaccuracies associated with the traditional presentation of these solutions in recommended sources will be shown.

Results and its discussion

Calculation of the electric field created by a uniformly charged ball of radius R with bulk density ρ

To solve this problem using the Maxwell integral equations, we need to use the following equation, which directly follows from the Gauss theorem after replacing the total charge inside the closed surface *S* by the corresponding integral

$$\oint_S E_n\, dS = \frac{1}{\varepsilon_0}\int_V \rho\, dV \qquad (1)$$

where: E_n is the projection of the electric field vector on the normal direction (unit vector $\vec{n}$, always directed outwardly to the closed surface *S*) to an infinitely small portion *dS* of this surface; *V* is the volume inside the surface *S*; *dV* is an infinitesimal volume allocated directly near the point at which the bulk charge density $\rho = dq / dV$; ε_0 is a constant associated with the SI system.

It is in this equation that the projection of the sought-for vector En is present (left) and the known quantity ρ (right).

Equation (1) is valid for any closed surface S. Therefore, when solving the problem, we have the right to choose this surface so that E_n can be taken out of the left integral. Obviously, this can only be done when $E_n = const$.

Therefore, before choosing the form of a closed surface of integration, it is necessary to establish a picture of the electrostatic field created by a given system of charges in the problem to be solved. More precisely, from some additional considerations, determine the course of the lines of force of the vector ***E*** in the entire space under consideration.

At the same time, it is not necessary to establish the direction of the power line.

For a number of symmetric charge distributions (spherical, cylindrical, uniformly charged plane) "this can easily be done using symmetry considerations," as written in all existing textbooks. The same "symmetry considerations" are recommended to be used when choosing the type of integration surface S. Unfortunately, nothing is written in the textbooks for such "symmetry considerations". Of course, a lot of experience, intuition and a large amount of residual knowledge can help the teacher solve these problems with the help of "symmetry considerations", especially without thinking about what it is.

But what about a student "gnawing at the granite of science with young teeth"?

The author believes that under the "use of symmetry considerations" in this context, the following actions should be understood [1]:

1. search for the geometrical place of points in the space under study, which are fundamentally indistinguishable,

2. the use of the "causality principle" for these points, i.e. statements that the same reasons should lead to the same consequences.

The latter inevitably leads to the conclusion that since these points are indistinguishable, then all the parameters of the desired force field at these points should be the same (otherwise they could be distinguished by these parameters).

We illustrate this with an example of solving the problem under discussion.

In Figure 1, a solid bold line shows a uniformly charged ball of radius R.

It seems obvious that in the absence of extraneous force fields, in addition to the electrostatic field of a uniformly charged (ρ = *const*) ball, all points lying on the surface of the ball are, in principle, indistinguishable. Therefore, the magnitude and direction of the vector E at all points of its surface should also be the same, and therefore directed at the same angle α to the normal vector to the surface n. However, there are no visible preferences for the magnitude and sign of this angle (α or - α).

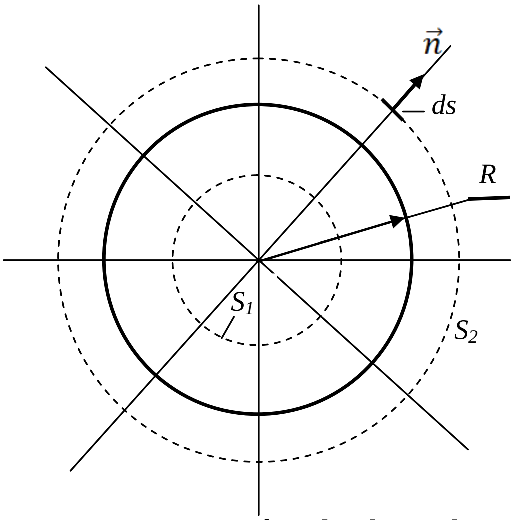

Figure 1. A uniformly charged ball of radius R

Taking into account that the vector ***E*** at each point of this surface can be directed in the only possible way with respect to the surface, it follows that $\alpha = 0$, and therefore, it is directed at the points under consideration perpendicular to the surface of the ball. The same can be said about the points of any spherical surface concentric with the surface of the ball of radius $r > 0$.

A similar conclusion can be reached mathematically.

Indeed, since all points lying on the surface of any concentric sphere are indistinguishable in our problem, the electric field potential at all these points must be the same (φ = *const*). Using the relationship between the vector ***E*** and φ (***E*** = - ***grad*** φ), it is easy to obtain [2-3] that the projection of the electric field vector into any direction tangent to this surface is $E_l = 0$. Therefore, the vector ***E*** is always perpendicular to the equipotential surface.

Therefore, if we choose a closed surface *S* in the formula (1) in the form of such a spherical surface, then it is obvious that at any point on this surface E_n = *const* (all points on this surface are indistinguishable, because they are at the same distance from the center, and therefore all the characteristics of the electric field at these points should be the same).

Figure 1 shows that all space by the surface of the ball naturally splits into two subspaces when $R \geq r > 0$ and when $r > R$.

In the first case, we can find E_n inside the ball. In the second - outside.

In the first case, from (1) for the surface s1 it follows

$$\oint_{S_1} E_n\, dS = E_n \oint_{S_1} dS = E_n 4\pi r^2 = \frac{1}{\varepsilon_0}\rho \int_V \; dV = \frac{\rho}{3} 4\pi r^3. \qquad (2)$$

Решая (2) относительно E_n , получим

$$E_n = \frac{\rho r}{3\varepsilon_0} \qquad (3)$$

From (3) it can be seen that for $\rho > 0$, $En > 0$, i.e. the vector ***E*** at all points of the surface S_1 is directed to the same place as the normal vector ***n***, i.e. outward of the surface S_1 . If $\rho < 0$, then $E_n < 0$, i.e. the vector ***E*** at all points of the surface S_1 is directed in the direction opposite to the normal vector ***n***. The vector form of the formula (3)

$$\vec{\boldsymbol{E}} = \frac{\rho r}{3\varepsilon_0}\vec{\boldsymbol{n}}. \quad (4)$$

It follows from (4) that the value of E in the center of the ball (r = 0) is equal to zero. This contradicts the widespread school view - "the thicker the power lines go, the greater the magnitude of the electric field strength". In the case of a point charge, this is correct, but our example shows that in the general case of a distributed charge, such a statement is completely untrue.

The concept of lines of force was introduced by Faraday. They helped him better pony-mother electric field of point charges. In fact, this is just a useful abstraction, helping to build the direction of the force vector at each point of the line directed along the tangent to it. And nothing more.

In the second case, as the integration surface in (1), we choose surface S_2 (Figure 1). Similarly, solving equation (1) and taking into account that the charge is located only in the volume of the ball of radius R), we obtain:

$$\oint_{S_2} E_n\,dS = E_n \oint_{S_2} dS = E_n 4\pi r^2 = \frac{1}{\varepsilon_0}\rho \int_V dV = \frac{\rho}{\varepsilon_0 3} 4\pi R^3. \quad (5)$$

Solving (5) with respect to En, we obtain for the electric field outside the ball

$$E_n = \frac{\rho R^3}{3\varepsilon_0 r^2}\,. \quad (6)$$

It can be seen from (6) that for $\rho > 0$, $E_n > 0$. i.e. the vector $\boldsymbol{E}$ at all points of the surface S_2 is directed to the same direction as the normal vector n, i.e. outward of the surface.

If $\rho < 0$, then $E_n < 0$, i.e. the vector $\boldsymbol{E}$ at all points of the surface S_2 is directed in the direction opposite to the normal vector n, i.e. can be written for this case in vector form:

$$\vec{\boldsymbol{E}} = \frac{\rho R^3}{3\varepsilon_0 r^2}\vec{\boldsymbol{n}}\,. \quad (7)$$

From (7) it is seen that the field of a uniformly charged ball outside the ball coincides with the field of a point charge equal to the charge of the ball placed in the center of the ball.

When deriving formulas similar to (3) and (6), a methodological error was made in [2]. Instead of the projection E_n, which can be both positive and negative, the value of the vector E is taken out of the sign of the integral. Therefore, instead of (3) and (6), in [2], respectively,

$$E = \frac{\rho r}{3\varepsilon_0} \text{ и } E = \frac{\rho R^3}{3\varepsilon_0 r^2}, \quad (8)$$

which is generally not true, because for $\rho < 0$, it turns out that the positive value of the vector E is less than zero.

Calculation of the magnetic field of an infinite solenoid with current I

The well-known university textbooks on the course of general physics, which are widely used when reading the physics course in the section "Electricity and Magnetism" for students of physical specialties, for example [2-7], argue that the magnetic field of an infinite solenoid with current I outside the solenoid equals zero.

Figure 2 shows a section of an infinite solenoid with current in this case, used to prove this statement in [2, 3].

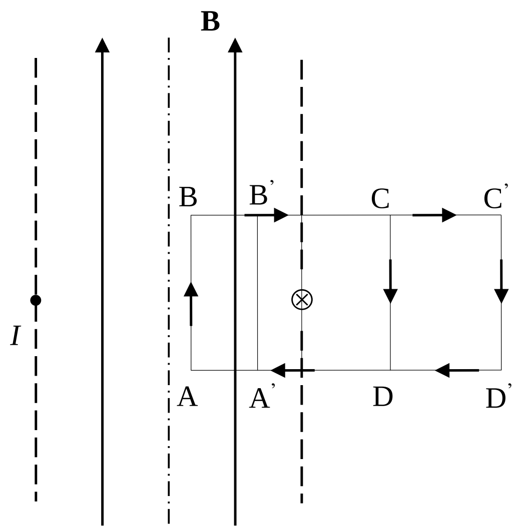

Figure 2 (from [3])

The authors consider the circulation of the vector ***B*** along the CC'D'D circuit and come to the conclusion that the value $B = 0$ outside the solenoid. This conclusion is really valid if the authors' assumption is true that the alleged direction of this vector is parallel and opposite to the direction of vector ***B*** inside the solenoid. However, this assumption needs proof, which is not given in [2, 3, 5 -7].

The author [4] tries to prove this by considering a solenoid in which the direction of the current in each turn is strictly perpendicular to the axis of the solenoid, i.e. the spiral nature of the current flow in the coil of the renal solenoid is not taken into account.

In this case, a real solenoid (Figure 3), any selected element ***dl*** with current *I*, in accordance with the Biot – Savart – Laplace law, will give the resultant vector ***dB*** at the point under consideration, the projection of which dB_l on the direction of the circle at the point under consideration, of radius r_0 equal to the distance from the point to the axis of the spiral, is no longer zero, and this must be taken into account when summing.

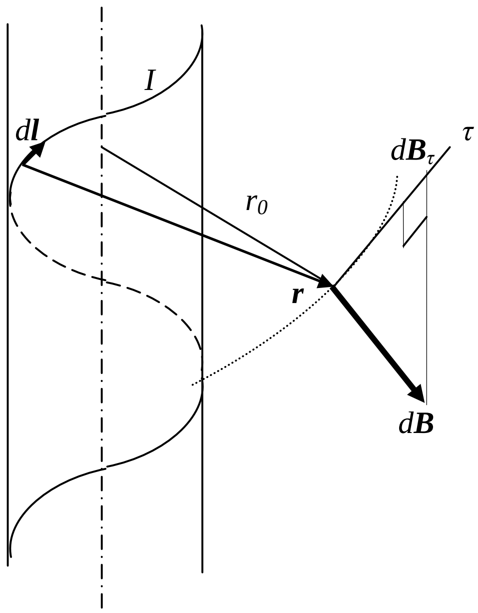

Figure 3. Vector ***dB*** generated by ***dl*** element current spiral I

Below, the author will try to correctly solve this problem using the corresponding Maxwell integral equation, without making additional assumptions.

We consider an ideal infinite solenoid with current *I* and radius *R*, which is tightly wound with a wire of such a cross section that makes the inner and outer surfaces of the solenoid smooth.

We will proceed from "symmetry considerations", by which the author understands what has already been discussed in solving the first problem.

In addition, we will take into account the fact that the magnetic field has no field sources, which is reflected in the closure of the lines of force of vector ***B***.

The geometry of the considered solenoid shows that all points lying on the surface of a cylinder of radius *r* coaxial with the axis of the solenoid are essentially indistinguishable. Therefore, the magnetic field induction vector B at all points on this surface should be directed identically with respect to the axis of the solenoid and its radius.

It is easy to see that this is done if the vector ***B*** is perpendicular to the radius of the solenoid and rotated by a constant angle relative to its axis. That is, in the general case of the vector ***B***, both inside the solenoid and outside it, it can take the form of a cylindrical spiral symmetrical about the axis of the solenoid (the causality principle does not prohibit this, since all points of this spiral are indistinguishable within the framework of the problem under consideration).

In this case, there is no question of the need to close this line, because the solenoid is infinite and has no "edges" on which this question would be relevant. Of course, there are possible, and for the same reasons, special cases of a spiral - a circle located in the plane perpendicular to the axis of the solenoid (the pitch of the screw is zero), and a straight line parallel to the axis of symmetry (the pitch of the screw is infinitely large).

The Maxwell equation necessary for solving the problem, containing the projection of the magnetic field induction vector ***B*** on the direction ***dl*** of the circuit *l* (B_l), in the case of direct current ($\partial\vec{E}/\partial t = 0$) has the form:

$$\oint_l B_l\, dl = \mu_0 \int_S j_n\, dS = \mu_0 I_l, \qquad (9)$$

where l is the contour (any closed line); S - any surface based on this contour (all points of the contour belong to this surface); ***j*** is the current density vector ($j_n=dI/ds$); I_l - total current covered by circuit l (flowing through surface S); μ_0 - is a constant associated with the SI system.

Obviously, B_l from (9) can be found if we choose the contour l in such a way that $B_l = const$.

Let us choose the rectangular contour l_1 , shown in Figure 4-a, as the integration contour.

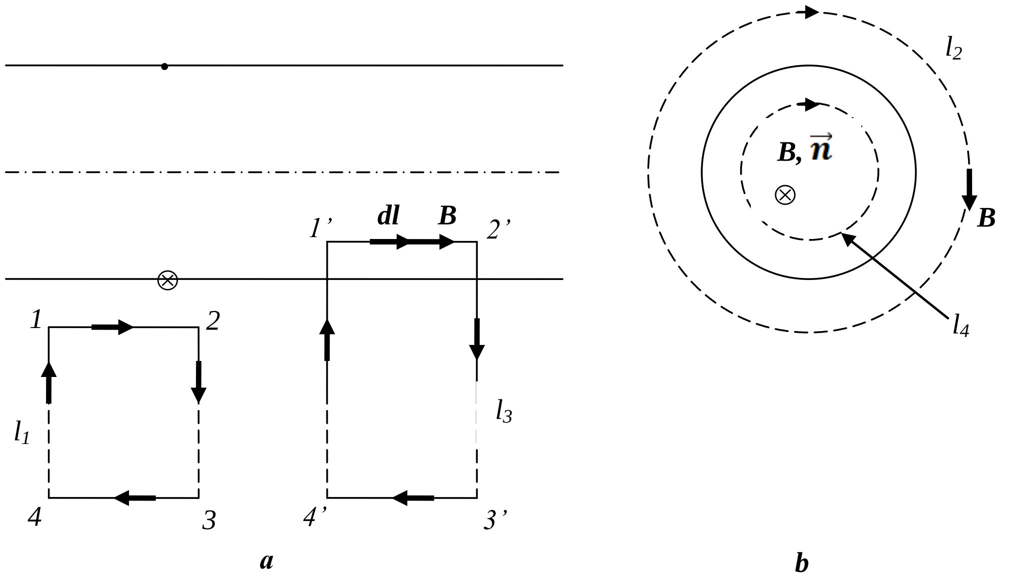

Figure 4. Correct calculation of the solenoid field
(Figure 4-b - left view of Figure 4-a)

In this case, with the correct consideration of the circulation of the vector ***B*** along the contour l_1, whose side 3.4 is removed to infinity, it follows from equation (9) that

$$\oint_{l_1} B_l\, dl = \int_1^2 B_l dl + \int_2^3 B_l dl + \int_3^4 B_l dl + \int_4^1 B_l dl \ = \mu_0 I_{l1} = 0 \ (I_{l1} = 0), \qquad (10)$$

where B_l is the projection of the vector ***B*** in the direction ***dl***; I_{l1} - total current covered by the circuit l_1.

At infinity (in sections 3-4), the solenoid field must obviously coincide with the forward current field, which, as is known [2], is inversely proportional to the distance to the current, i.e. ***B*** = 0 in this section, and therefore the integral in this section is equal to zero.

In sections 2-3 and 4-1, the tangent to the spiral at any point is perpendicular to ***dl***. Therefore, $B_l = 0$, and the integrals on these sections are also equal to zero. From the symmetry of the problem it follows that in section 1-2 all points are indistinguishable, and therefore $B_l = const$, which can be taken out of the sign of the integral.

Therefore, $B_l = 0$, i.e. the projection of the vector ***B*** on the direction $l_{1,2}$ is zero, and not the magnitude of the vector ***B***, as stated in [2-7].

It remains to check whether the line of force of the vector ***B*** is a circle drawn around the axis of symmetry of the solenoid, which does not contradict the "symmetry considerations", because it is obvious that all points on this circle are indistinguishable, and therefore the projection of the vector B_l at all points of the contour l_2 in Figure 4-b, selected in the form of such a circle will be a constant value. Therefore, applying the Maxwell equation for this circuit [14], we obtain:

$$\oint_{l_2} B_l\, dl = B_l \oint_{l_1} dl = B_l 2\pi r = \mu_0 I_{\text{охв}} = \mu_0 I. \qquad (11)$$

Откуда следует, что

$$B_l = \frac{\mu_0 I}{2\pi r}. \qquad (12)$$

In accordance with the rule for choosing the sign of current I, its value in (11) is taken with the sign "+" if the direction of current flow coincides with the direction of the normal to the surface stretched on the circuit l_2. The direction of the normal (ort $\vec{n}$) is determined by the direction of movement of the right screw, which must be rotated in the direction of the selected direction of the circuit bypass.

In our case (Figure 4-b), the selected direction of the circuit traversal gives the direction of the unit normal vector нормали $\vec{n}$ indicated in the figure, which coincides, with the direction of the current I flowing through the surface stretched over the circuit l_2, i.e.

$$B_l > 0; \Rightarrow B_l = B = \frac{\mu_0 I}{2\pi r}, \qquad (13)$$

and therefore, the vector ***B*** is directed outside the solenoid as indicated in Figure 4-b.

With a correct consideration of the solenoid field inside the solenoid in the same problem, it is also necessary to take into account a priori that a possible helix of the vector ***B*** inside the solenoid is also a similar helical line passing inside the solenoid. Therefore, applying equation (9) for circuit l3 (Figure 4-a), taking into account output (10), we obtain:

$$\oint_{l_3} B_l\,dl = \int_{1'}^{2'} B_l dl + \int_{2'}^{3'} B_l dl + \int_{3'}^{4'} B_l dl + \int_{4'}^{1'} B_l dl = \int_{1'}^{2'} B_l dl =$$

$$= B_l l_{1'2'} = \mu_0 I_{\text{охв}} = \mu_0 n\, l_{1'2'} I\,, \qquad (14)$$

where n is the number of turns per unit length of the solenoid ($n = N_{1'2'}/l_{1'2'}$).

Solving (13) with respect to B_l , we obtain that on the segment $l_{1'2'}$

$$B_l = \mu_0 n I. \qquad (15)$$

Similarly, applying (9) for the contour l_4 (Figure 4-b), we obtain that for this contour $B_l = 0$.

Then, applying the sign rule for the currents covered by the l_3 contour, we finally obtain the correct answer for the magnitude of the vector ***B*** inside the solenoid

$$B = \mu_0 nI. \qquad (16)$$

In textbooks [2, 4-7], it is stated that the magnetic field of a toroid with a current outside the toroid is zero. In support of this, in [2], the circulation of the vector B is considered along any circular contour l lying outside the toroid whose plane is perpendicular to the axis of symmetry of the toroid and the center of the contour lies on its axis.

In this case, circulation along this circuit since such a circuit does not cover currents.

$$\oint_l B_l\,dl = B_l \oint_l dl = 0, \qquad (17)$$

It follows that $B_l = 0$, from which the author concludes that the vector ***B*** = 0 at any point outside the toroid. This conclusion is based on the assumption that the direction of this vector at any point in the contour *l* is parallel to the vector ***dl.***

The fallacy of such an assumption for an infinite solenoid was shown above.

From formula (17) it follows only that the desired vector ***B*** does not lie in the plane of the contour *l*. And you should check the possibility of its orientation at any point outside the toroid with current I in the plane perpendicular to the axis of the toroid, that is, find the circulation of the vector ***B*** along any contour l_1 lying in the plane perpendicular to the axis of the toroid (Figure 5).

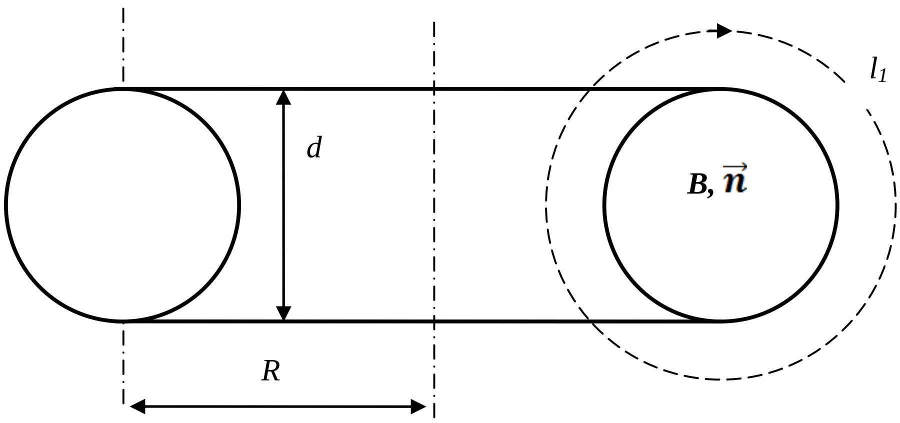

Figure 5. To the calculation of the magnetic field of a toroid with a current *I* outside the toroid

Since the surface resting on the circuit l_1 crosses the conductor with current *I* only once, this circulation is equal to

$$\oint_{l_1} B_l\,dl = \mu_0 I_{\text{охв}} = \mu_0 I > 0 \;\Rightarrow\; B_l > 0\,, \qquad (18)$$

It follows from formula (18) that the value $B_l > 0$ at all points of the contour at located outside the toroid.

In the case *R* >> *d*, the value of the induction vector ***B*** inside the toroid will be close to the value [2]

$$B = \mu_0 n I, \qquad (19)$$

coinciding in magnitude and direction with the field of an infinite solenoid inside the solenoid.

Taking into account that at distances much larger than *R*, the field of such a toroid with tight winding should coincide with the field of a flat coil with a current of a coil of radius R, as well as the calculation results for the magnetic field of the solenoid outside the solenoid, carried out above, there is every reason to consider that the field of any toroid outside it will coincide with the field of a flat turn with current *I*.

But in no case is it equal to zero, as stated in almost all textbooks known to the author [2, 4-7].

Main conclusions

1. Used in solving symmetric problems to establish the picture of the electric field and magnetic field, the so-called "symmetry considerations" are a reflection of the well-known causality principle, namely, the same causes should lead to the same consequences. This principle is applied after finding the geometrical location of indistinguishable points at which all parameters of the desired field should be identical.

2. Using Maxwell's equations in integral form, we obtained the correct solution of two well-known symmetric problems. It is shown that in order to avoid methodological and factual errors, it is impossible to replace the projection of a vector with its value, because the projection may have a sign, and the value of the vector is always positive. In addition, equality to zero of the projection of the vector does not mean equal to zero of the vector. It was such a substitution that led to the wrong answer when finding the induction of the magnetic field of the solenoid outside the solenoid, which has been translated for more than 40 years from textbook to textbook.

3. It is shown that the magnetic field of the direct infinite solenoid outside the solenoid in the indicated approximations is not equal to zero, but coincides with the field of the direct infinite conductor with current.

4. It is shown that the magnetic field outside the toroid is also non-zero, and arguments are also presented in favor of the fact that this field coincides with the field of a flat turn with current.

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