

Increased contact time of the sulfuric acid solution and calcine may increase impurities in the solution. When calcine leaching with sulfuric acid react primarily oxidized minerals zinc and copper. Minerals of iron and silver react with sulfuric acid slowly. Therefore, in order to achieve maximum extraction of zinc with minimal impurities moving in the solution, leaching time can be installed 2 hours

Speed of vast majority of chemical reactions, and also the diffusion increase with rise of temperature. With a rise of temperature there is a slow increase in the concentration of zinc in the solution. However, starting from 40°C with an increase in the duration of the process there is more intensive increase in the degree of extraction of zinc and copper. This is because at high temperatures ZnSO<sub>4</sub> is formed more rapidly. It is expected that with further increase in temperature will increase the rate of dissolution. At the same time it is necessary to consider that increasing the temperature significantly affects on the dissolution of the useful component (the concentration of zinc), while the transition of the impurity into the solution is greatly increasing. Increasing the temperature over 80°C has little effect on leaching of the concentration of zinc, but highly increases the transfer of impurities into the solution. The required hydrodynamical mode to achieve a homogeneous slurry density provided with a mechanical stirring device.

Thus, the following optimal conditions for leaching zinc cake after thermo-steaming were set: the sulfuric acid concentration of 125-150 g / l, 75-800S temperature, duration 2 h. In these conditions the degree of extraction of zinc into into the solution is 85-95% and iron is 28.1%, and the yield of cake is 58-60% of the calcine weight. Results of the study indicate the possibility of efficient processing of zinc cakes using the method of thermo-steaming followed by sulfuric acid leaching.

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## MODELLING OF A THREE-DIMENSIONAL PROBLEM OF DISTRIBUTION OF HARMFUL IMPURITY IN THE RIVER A RECURRENTLY-OPERATIONAL METHOD

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#### ABSTRACT

In this article the decision of a three-dimensional problem of diffusion is considered by a recurrently-operational method which describes process of distribution of harmful impurity along a watercourse.

The received numerical results on the COMPUTER where it is possible to define for what time are resulted there is a distribution and river clarification. The received results are illustrated in drawings.

**Keyword:** Modelling, process of distribution of harmful impurity, recurrent parity, the recurrent equation, diffusion factor, factor no conservation, the exact decision, Problem Kashi, concentration of emission.

#### Introduction

With the growth in the development of industrial enterprises, emissions of harmful substances into the atmosphere and the water environment increase, along

with this, with an even increase in production, land is depleted, improper use of chemical fertilizers, various harmful emissions significantly affect water and land resources.

With the industrial effluent of enterprises, a certain number of different substances enter the rivers, the variety of which increases. In this regard, it is most rational to conduct an integrated assessment of pollution by generalized hydrochemical characteristics of water quality: weighing of the substance, biochemical oxygen consumption, toxic meteorology of the sign of harmfulness[1,2 p.12-15].

In many monographs the decision of these models are given only with use of numerical methods and difference schemes, thus there is some question, the methods of the decision of various problems connected with a choice on which depend a practical realizability, accuracy and duration of reception of the decision on the computer.

In research of a problem of atmospheric diffusion and environmental contamination the huge contribution was brought by scientists - mathematics and mechanics G. I. Marchuk, M. E. Berljand, V. K. Kabulov, F. B. Abutaliev, S. Karimberdieva, M. A. Vladimirov, J. I. Ljahin, L. T. Matveev, V. G. Orlov, YU. V. Shokin, V.

M. Belolipetsky, G.Ivahnenko, Y A.Muller, A.E.Alojan, V.V. Penenko, Yu. V. Koppa, A. N. Groshkov, P.N.Belov, and K.I.Kachiashvili, D. G.Gordeziani, D.I.Melikdzhanchan many other things.

**Mathematical statement of a problem.** Let's consider a three-dimensional problem of diffusion describing process of carrying over of polluting substances in river water [4, p 42] in a kind:

$$\frac{\partial q}{\partial t} = k_x \frac{\partial^2 q}{\partial x^2} + k_y \frac{\partial^2 q}{\partial y^2} + k_z \frac{\partial^2 q}{\partial z^2} - v \frac{\partial q}{\partial x} - \alpha q + f \quad ; (1)$$

We solve the given equation in the absence of a source of emission of harmful impurity, that is the homogeneous equation at ( $f = 0$ ).

Having divided the equation (1) on factor  $k_x$  by  $\frac{\partial^2 q}{\partial x^2}$  we will copy in a kind

$$\frac{\partial^2 q}{\partial x^2} = -a_1 \frac{\partial^2 q}{\partial y^2} - a_2 \frac{\partial^2 q}{\partial z^2} + a_3 \frac{\partial q}{\partial x} - a_4 \frac{\partial q}{\partial t} + a_5 q \quad (2)$$

where  $a_1 = \frac{k_y}{k_x}; a_2 = \frac{k_z}{k_x}; a_3 = \frac{v}{k_x}; a_4 = \frac{1}{k_x}; a_5 = \frac{\alpha}{k_x};$

**Decision method.** For the decision the equation (1) we search in the form of a number [5,6. p 158, 184] The decision of the equation (1) is searched in a kind

$$q = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{p=0}^{\infty} Q_{i,j,k,p} x^{i+j+k+p+r!} \partial_y^j \partial_z^k \partial_t^p g(y, z, t) \quad (3)$$

Substituting the decision (3) in (2), we receive a following recurrent equation:

$$Q_{i,j,k,p} = -a_1 Q_{i,j-2,k,p} - a_2 Q_{i,j,k-2,p} + a_3 Q_{i-1,j,k,p} + a_4 Q_{i-1,j,k,p-1} - a_5 Q_{i-2,j,k,p} \quad (4)$$

At entry conditions  $Q_{0,0,0,0} = 1, Q_{i,j,l,p} = 0$ , при  $i < 0, j < 0, k < 0$  или  $p < 0$  (5)

$$\begin{aligned} Q_{0,0,0,0} &= 1, Q_{1,0,0,0} = a_3; Q_{0,1,0,0} = 0; Q_{0,0,1,0}; Q_{0,0,0,1} = 0; \\ Q_{2,0,0,0} &= a_3^2 + a_5; Q_{0,2,0,0} = -a_1; Q_{0,0,2,0} = -a_2; Q_{0,0,0,2} = 0; Q_{1,1,0,0} = 0, \\ Q_{1,0,1,0} &= 0; Q_{1,0,0,1} = a_4; Q_{0,0,1,1} = 0; Q_{0,1,1,0} = 0; Q_{0,1,0,1} = 0; \\ Q_{3,0,0,0} &= a^2 a_3 + 2a_3 a_5; Q_{0,3,0,0} = 0; Q_{0,0,3,0} = 0; Q_{0,0,0,3} = 0; Q_{2,1,0,0} = 0; Q_{2,0,1,0,0} = 0; \\ Q_{2,0,0,1} &= a_3 a_4; Q_{0,2,1,0} = 0; Q_{0,2,0,1} = 0; Q_{1,2,0,0} = -2a_1 a_3; Q_{1,0,0,2} = 0; Q_{0,1,2,0} = 0; Q_{0,0,1,2} = 0; \\ Q_{0,1,0,2} &= 0; Q_{1,1,1,0} = 0; Q_{1,1,1,0} = 0; Q_{1,0,1,1} = 0; Q_{1,1,0,1} = 0; Q_{0,1,1,1} = 0. \end{aligned}$$

Writing out some first members of some (3), we have

$$\begin{aligned} q(t, g(x, y, z)) &= g_0 + [a_3 g'_x] x + [-a_2 g''_z - a_1 g''_y + a_4 g'_x g'_t] x^{2,1} + \\ &+ [a_3(a^2 + 2a_5) g'''_x + a_3 a_4 g''_x g'_t + (-2a_1 a_3) g'_x g''_y + (-2a_2 a_3) g'_x g''_z] x^{3,1} + \dots \quad (6) \end{aligned}$$

The function, satisfying to entry conditions at  $t_0 = 0.1$ , following:

$$. g(x, y, z, t) = C e^{\left( \frac{(x-vt_0)^2}{4k_x t_0} - \frac{y^2}{4k_y t_0} - \frac{z^2}{4k_z t_0} - \alpha_0 \right)} \quad (7)$$

Where  $C = \frac{N}{2w\sqrt{\pi k_x k_y k_z t_0}}$

Substituting (7) in (6), we have

$$g(t, g(x, y, z)) = C e^{-\left(\frac{(x-a_1 t_0)^2}{4a_2 t_0} + a_{00} t_0\right)} \left( \left[ a_3 \frac{(x-vt_0)}{2k_x t_0} \right] x + \left[ a_2 \frac{z}{2k_z t_0} + a_1 \frac{y}{2k_y t_0} + a_4 \frac{-x(x-vt_0)}{4k_x t_0^2} + \frac{y^2}{4k_y t_0^2} + \frac{z^2}{4k_z t_0^2} \right] x^{2,1} + \dots \right) + \dots$$

This number is turned off in function

$$g(x, y, z, t) = \frac{N}{2w\sqrt{\pi k_x k_y k_z t_0}} e^{-\left(\frac{(x-vt_0)^2}{4k_x t_0} - \frac{y^2}{4k_y t_0} - \frac{z^2}{4k_z t_0} - \alpha t_0\right)}$$

Consider the transfer of contaminants in the Choga river section between sections 1 and 2 using a three – dimensional model. The characteristic data of the river used to model water pollution are [4, 42 p.].

Equation (1) is solved under certain initial and boundary conditions, the instantaneous point source of a unit mass of pollutant, and the initial condition  $q(x, y, z, 0) = 0$

The initial condition  $q(x, y, z, t)|_{t=t_0} = C e^{-\left(\frac{(x-vt_0)^2}{4k_x t_0} - \frac{y^2}{4k_y t_0} - \frac{z^2}{4k_z t_0} - \alpha t_0\right)}$

$$q(x, y, z, 0) = q_0(x, y, z), 0 \leq x \leq l_1, 0 \leq y \leq l_2, 0 \leq z \leq l_3,$$

Boundary conditions  $q(0, y, z, t) = q_0(y, z, t); 0 \leq z \leq l_3; 0 \leq y \leq l_2; 0 \leq t \leq T;$   
 $l_1$  – averaged length of the watercourse,  $l_2$  – averaged width,  $l_3$  – depth.

Where  $C = \frac{N}{2w\sqrt{\pi k_x k_y k_z t_0}}$ ;  $w$  – area of a live section.

$$\frac{\partial q(x, y, z, t)}{\partial x} |_{x=l_1} = 0, 0 \leq z \leq l_2, 0 \leq y \leq l_2, 0 \leq t \leq T;$$

$$\frac{\partial q(x, y, z, t)}{\partial y} |_{y=0} = \frac{\partial q(x, y, z, t)}{\partial y} |_{y=l_2} = 0, 0 \leq x \leq l_1, 0 \leq z \leq l_3, 0 \leq t \leq T;$$

$$\frac{\partial q(x, y, z, t)}{\partial z} |_{z=0} = \frac{\partial q(x, y, z, t)}{\partial z} |_{z=l_3} = 0, 0 \leq x \leq l_1, 0 \leq y \leq l_3, 0 \leq t \leq T;$$

The results are obtained by the recurrent – operator method for the three – dimensional problem of the spread of harmful impurities and the diffusion of river pollution.

**Discussion of results.** Being set as  $g^*(t, g(x, y, z))$  values  $t = t_1, t = t_2, t = t_3, \dots$ , we build the combined schedule of function  $g_1^*, g_2^*, g_3^*, \dots$

In a recurrently-operational method the decision turns out in the form of (3), and at Kachaishvili K. I., Gordesiani D. G., Melikzhanyan G. I. it is received

$$g(x, y, z, t) = \frac{N}{2w\sqrt{\pi t_0 k_x k_y k_z}} e^{-\left(\frac{(x-v(t_0+t))^2}{4k_x(t_0+t)} - \frac{y^2}{k_y t_0} - \frac{z^2}{k_z t_0} - \alpha t_0\right)}$$

If two decisions of a different kind, in this case (7), satisfy to the same differential equation (1) and to same entry conditions  $t_0$  under Sofia Kovalevskoj's theorem of uniqueness of the decision of problem Kashi these both decisions coincide (i.e. schedules of these functions are identical).

**Conclusions.** Results of the decision of the equations of emission of harmful impurity at the moment of time with use of a recurrently-operational method are resulted in table 1.

Table 1

**Modelling of process of distribution of harmful impurity during the initial moment of time**

n	t,s	x, m	y, m	z,m	qg/m <sup>3</sup> s
1	0.1	1	1	0.5	0.452717638135
2	0.1	2	1.5	0.5	0.297292612372
3	0.1	2	2	1	0.102037281694
4	0.1	2	3	1	0.027526784011

Below in figures 1, 2 the results of modeling the distribution of the emission of harmful impurities at the initial moment of time are shown.

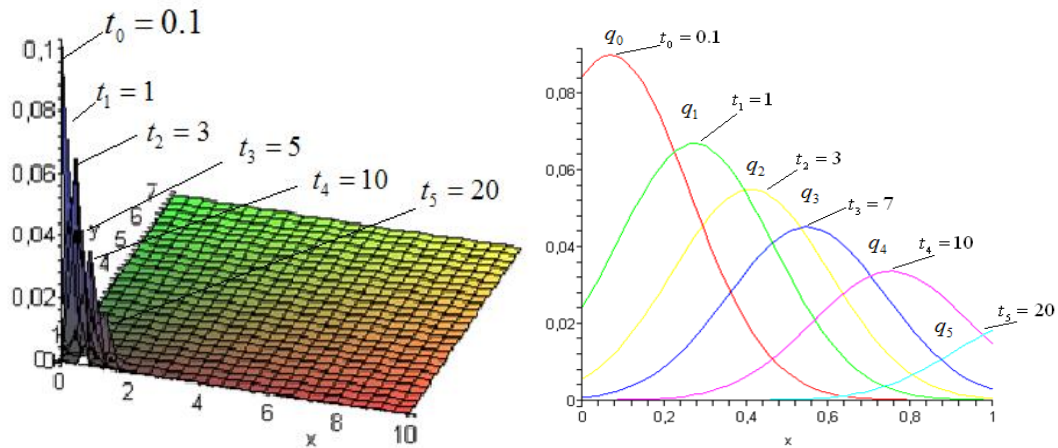


Figure 1. At the initial moment of Figure 2. The process of emission time impurities with the passage of time of harmful along the axis OX

Next in tables 2-4 are the results of modeling the spread of harmful impurities and diffusion at the boundary conditions Border conditions:

Table 2

**Modelling of process of distribution of harmful impurity on border at**

n	t, s	x,m	y,m	z,m	qg/m <sup>3</sup> s	x,m	y,m	z,m	qg/m <sup>3</sup> s
1	1	0	0	0.5	0.195484191468	10	0	0	0.549857533898
2	3	0	1	0.5	0.054738798301	10	1	0.5	0.0988982444101
3	5	0	1	1	0.054434883807	10	1.5	1	0.0217111310508
4	7	0	1.5	1.5	0.008779613951	10	3	1.5	0.0070254965233
5	10	0	3	1.5	0.002532292528	10	5	2	0.0016770259872
6	20	0	5	2	0.0000053523774	10	7	3	0.00000298429029

Table 3.

**Modelling of process of distribution of harmful impurity on width of the river on border at**

n	t, s	x,m	y,m	z,m	qg/m <sup>3</sup> s	y,m	z,m	qg/m <sup>3</sup> s
1	1	1	0	0.5	0.110777148418	7	0.5	0.0378935847457
2	3	2	0	1	0.016097099822	7	1	0.01122394935932
3	5	3	0	1.5	0.0032284949531	7	1.5	0.00257820441627
4	7	5	0	1.5	0.0011962795528	7	1.5	0.00235472244768
5	10	7	0	3	0.00007524547040	7	3	0.00231170414418
6	20	10	0	3	0.000003173047747	7	3	0.00000298429029

Table 4.

Modelling of process of distribution of harmful impurity on depth of a waterway							
n	$t, s$	$x, m$	$y, m$	$z, m$	$qg/m^3s$	$z, m$	$qr/m^3c$
1	0.1	0	0	0	0.852304144067796	3	0.11484963135593
2	1	1	1	0	0.108983968248587	3	0.08917935271186
3	3	2	1	0	0.016099224716949	3	0.01504963230790
4	5	3	1.5	0	0.00322909308146	3	0.003096125297740
5	7	5	3	0	0.00116548616071	3	0.001124684456485
6	10	7	5	0	0.00072117137687	3	0.000703662604192
7	20	10	7	0	0.000030187133123	3	0.000029842902997

In Figures 3 – 4 show the results of modeling the spread of harmful impurities and diffusion over time.

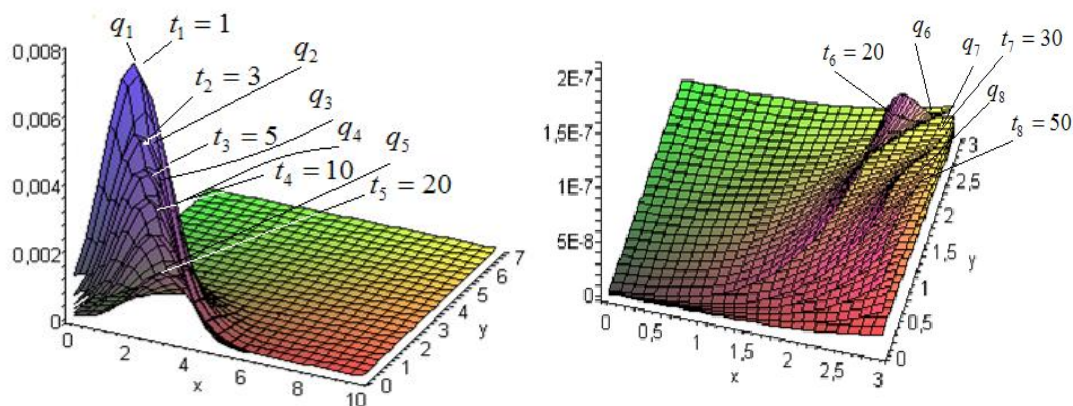


Figure 3. The process of spreading harmful impurities in time  
Figure 4. The process of spreading harmful impurities over time

**Acknowledgements** The obtained results coincide with the results of the work of other authors by a recurrent – operator.

According to fig. 4, over time, the intensity of emission of harmful impurities decreases and the concentration of emission of harmful impurities reaches the maximum allowable emission rate for 13 min. 30 sec.

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