ТЕХНИЧЕСКИЕ НАУКИ

THE APPLICATION OF SUPER RESOLUTION ALGORITHMS IN PHASED ARRAY ANTENNA FOR SMALL MULTI-CHANNEL DF DEVICES

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ABSTRACT

Direction finding (DF) has always been an important area, especially in passive systems where direction finding is the key to solve important technical problems. This task is often much more difficult in small DF devices where the size and weight of the antenna are limited. In this case, the limitation of the antenna dimension must be compensated by the high resolution DF algorithms, advanced antenna designs, and powerful computation performance.

This paper proposes the use of super resolution algorithms in small phased array antenna for multi-channel DF devices. This research could result in the optimization in size of DF devices, based on super resolution algorithms.

Keywords: DOA, estimation, resolution, direction finding, phased array antenna.

Introduction

The accuracy of the DF depends on the antenna beamwidth, θ, as follow:

$$\theta = k\frac{\lambda}{D} \quad (1)$$

In that:

λ : the wavelength;

D: the antenna dimension;

k: scale factor, depending on the amplitude distribution of the antenna.

In order to increase the accuracy and resolution, it is necessary to decrease the antenna beamwidth. This can be done by either reducing the wavelength or extending the antenna size. However, reducing the wavelength is equivalent to increasing operational frequency, which is not always possible because the operational frequency is an important parameter of the system and depends on many other factors. Similarly, the increase of the antenna size normally results in the raise of mass, cost and the reduction of mobility which are not always desirable in real applications. Therefore, if the frequency band is fixed, a preferable solution for the small multi-channel DF devices is to use super resolution algorithms in accordance with phased array antenna.

DF super resolution algorithms are mentioned in [1, 3, 5], each has its own advantages and disadvantages, and can be applied in specific phased array antenna configurations. In the limit of this paper, the Capon, EV and MUSIC algorithms are evaluated in terms of performance. The three algorithms are not only simple and easy to compute but also can be applied to arbitrary phased array antenna shapes. Those algorithms, therefore, are especially suited for small multi-channel DF devices where the mobility of the system is an priority.

In this paper, two main antenna configurations in DF systems are investigated using the above super resolution algorithms. They are the circular antenna and uniform linear antenna. The 2 antenna configurations are simple, yet they are highly flexible in DF for different directions and in different planes.

The remainder of this paper is organized as follows: section II reviews DF super resolution algorithms and phased array antenna, section III investigates the dependence of the DF performance on signal processing algorithms and antenna configurations. Finally, the conclusion is given in Section V.

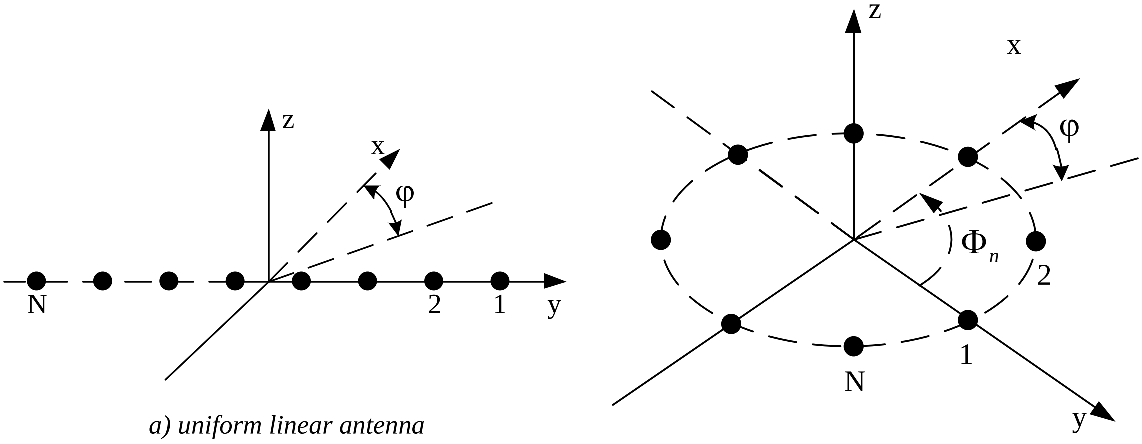
A review of DF super resolution algorithms and phased array antenna

Figure 1 shows the configuration of phased array antennas which can be used in multi-channel DF devices: a – uniform linear antenna in which the antenna's elements are evenly spaced and b – circular antenna. The antenna parameters are:

N: number of antenna elements;

$\varphi_k, k = \overline{1, M}$: the angle of arrival of signal;

Φ_n: the angle between the first and the nth antenna elements in a circular antenna, and so $\Phi_n = n\frac{2\pi}{N}$, $n = 1,2 \ldots N$.



a) uniform linear antenna

b) circular antenna

Figure 1. the configuration of phased array antennas in multi-channel DF devices

Assume that there are M sources of signal in the far field area with the azimuth $\varphi_k, k = \overline{1, M}$and the waves from those sources of signal to N antenna's elements are plane waves. The vector of signal's samples received at time t can be expressed as [3]:

$$x(t) = As(t) + n(t) \qquad (2)$$

Where:

$x(t)$: signal vector received by the phased array antenna at time t, with size $N \times 1$;

$s(t)$: signal vector at time t, with size $M \times 1$;

$n(t)$: noise vector at time t, with size $M \times 1$;

A: matrix of steering vectors of signal sources, with size $N \times M$.

Different phased array antenna configurations have different steering vectors, such as [2,4]:

– For the uniform linear antenna: $a(\varphi) = \left[1 e^{-j\frac{2\pi d}{\lambda} sin(\varphi)} \ldots e^{-j\frac{2\pi d}{\lambda}(N-1)\, sin(\varphi)}\right]^T$;

– For the circular antenna: $a(\varphi) = \left[1 e^{-j\frac{2\pi r}{\lambda} cos\left(\varphi - \frac{2\pi}{N}\right)} \ldots e^{-\frac{j2\pi r}{\lambda} cos\left[\varphi - (n-1)\frac{2\pi}{N} - \pi\right]}\right]^T$.

In that:

λ: the wavelength;

d: the distance between 2 adjacent elements of the uniform linear antenna;

r: the radius of the circular antenna.

The DF of all the three super resolution algorithms are based on the maximum in the pseudospectrum in the signal space $P(\varphi)$. However, the ways to calculate the pseudospectrum in the signal space are different.

+ For the CAPON algorithms, the pseudospectrum in the signal space is calculated as follow:

$$P_{CAPON}(\varphi) = \frac{1}{a^H(\varphi) R^{-1} a(\varphi)} \qquad (3)$$

In that R is the correlation matrix of input signals received by antenna's elements and is determined by $R = XX^H$

Matrix R has eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N$ and can be divided into 2 sub-matrices:

-Signal sub-space matrix $Q_s = [e_1, e_2, \ldots, e_M]$: this is a matrix of eigenvectors corresponding to the M largest eigenvalues.

- Noise sub-space matrix $Q_s = [e_1, e_2, \ldots, e_M]$: this is a matrix of eigenvectors corresponding to the N-M smallest eigenvalues $\Lambda_n = [\lambda_{M+1}, \lambda_{M+2}, \ldots, \lambda_N]$.

+ The MUSIC algorithm calculates the pseudospectrum in the signal space as follow [5, 6]:

$$P_{MUSIC}(\varphi) = \frac{1}{a^H(\varphi) Q_n Q_n^H a(\varphi)}. \qquad (4)$$

+ The EV algorithm is different from the MUSIC algorithm in a way that it calculates the eigenvalues corresponding to noise sub-space. In that, the pseudospectrum in the signal space is calculated as follow:

$$P_{EV}(\varphi) = \frac{1}{A^H(\varphi) Q_n \Lambda_n^{-1} Q_n^H A(\varphi)}. \qquad (5)$$

The purpose of this paper is to design the DF solutions for multi-channel DF devices which not only meets the DF parameter requirements but also small and highly mobile. In order to do so, the signal processing method and antenna configuration need to be optimized based on the DF parameter requirements such as DF accuracy, antenna size, signal processing capability, real time display capability. In the next section, we will examine the dependence of the DF capability on different factors such as signal processing algorithms or antenna configurations.

The dependence of the DF capability on signal processing algorithms and antenna configurations

In this sections, let us consider the 2 most common antenna configurations for the simulations of the Capon, EV and MUSIC algorithms. They are the uniform linear and circular antenna, each has 8 elements. The DF requirements are:

- Requirement 1: RMSE is not greater than 1 degree;

- Requirement 2: number of wrong DOA estimates is not greater than 1% of measured samples. A wrong DOA estimates is counted when the difference between the estimated AOA and the real AOA is greater than 1 degree;

- Requirement 3: The DF resolution is no worse than 1 degree. Note that in order to differentiate the AOA of 2 adjacent signal sources, in the pseudospectrum the peaks corresponding to the AOA of the 2 signals must be 1-3dB higher than the trough between them.

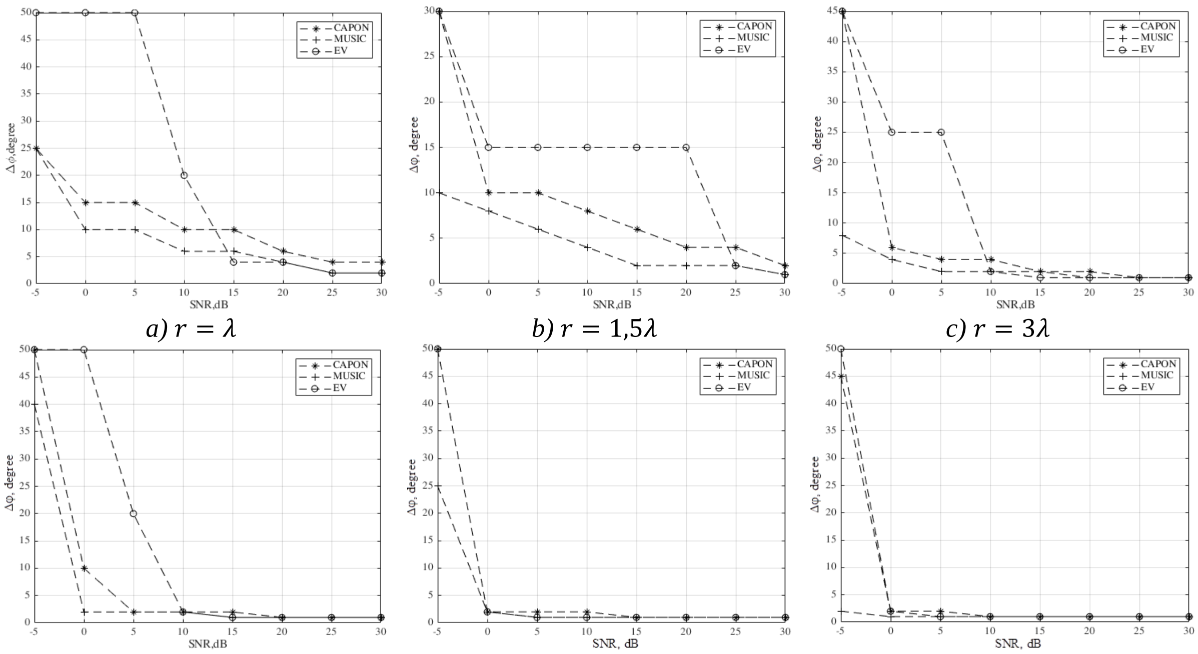
Among the three above requirements, the first two can be achieved easily in most DF devices. The third requirement, on the other hand, is the most difficult one and always a challenge in DF devices.

All the simulation in this section is implemented in Matlab. The simulation scenario is set up as the following: the number of target is known and smaller than the number of elements in the phase array; noise introduced into the simulation is the White Gaussian Noise. Other parameters in the simulation are: SNR, number of snapshot N_x, and antenna sizes.

<u>The dependence of the DF resolution on the radius of the circular antenna</u>

The purpose of this section is to find the optimum radius of the circular antenna so that the DF device not only meet the DF requirements but also requires the least SNR.

Figure 2 shows the dependence of the DF resolution ($\Delta\varphi$) of all 3 algorithms, using the circular antenna with 8 elements, on the radius of the circular antenna. In the simulations, the SNR changes from -5 to 30dB, which covers full SNR range in DF devices. The number of snapshot is $N_x = 200$ which can guarantee a reasonably high DF performance but does not cause significant computation requirement.



a) $r = \lambda$ *b) $r = 1{,}5\lambda$* *c) $r = 3\lambda$*

d) $r = 4\lambda$ *e) $r = 5\lambda$* *f) $r = 6\lambda$*

Figure 2. The dependence of the DF resolution on the radius of the circular antenna

From the results in Figure 2, several conclusions can be drawn as follows:

- For the phase array circular antenna, the increase in the radius can compensate for the decrease in the SNR;

- Figures 2a, 2b, 2c show that if the radius of the phase array circular antenna is smaller than 4λ, the SNR need to be no less than 20dB to meet the DF resolution requirement;

- From Figures 2d, 2e, 2f, if the radius of the phase array circular antenna is greater or equal to 4λ, the DF resolution requirement can be met even with the SNR is about 10-15dB. If we keep increasing the radius, the required SNR is decreased but slowly. The optimum radius for the circular antenna is, therefore, around $\boldsymbol{4\lambda}$.

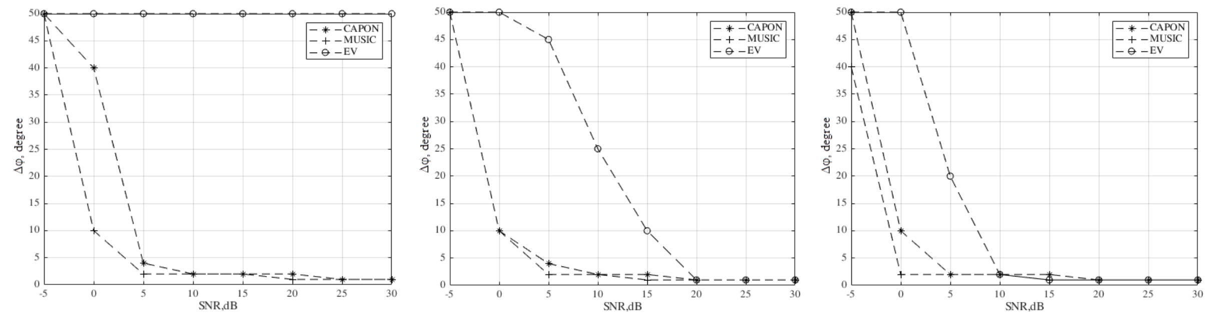
- Among those algorithms, the MUSIC algorithm requires the lowest SNR.

<u>The dependence of the DF resolution on the number of snapshot</u>

In this section, let us consider 2 antenna configurations: uniform linear antenna with $d = \lambda/2$ and circular antenna with radius $R = 4\lambda$, both have 8 elements. The radius of the circular antenna is chosen as 4λ based on the result of the subsection 3.1.

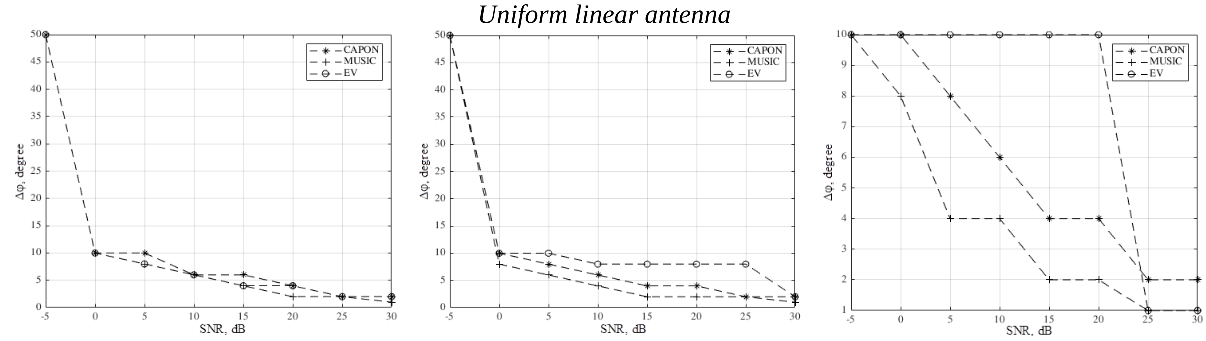
Figure 3 shows the dependence of DF resolution on SNR with different numbers of snapshot, for Capon, MUSIC and EV algorithms. Figures 3.a, 3.b and 3.c are the results of the circular antenna while Figures 3.d, 3.e and 3.f are the results of the uniform linear antenna.

Uniform circular antenna



a) $N_x = 50$ *b)* $N_x = 100$ *c)* $N_x = 200$

Uniform linear antenna



d) $N_x = 50$ *e)* $N_x = 100$ *f)* $N_x = 200$

Figure 3. The dependence of the DF resolution on the number of snapshot

From the results in Figure 3, several conclusions can be drawn as follows:

- With the same number of snapshot, the circular antenna with radius 4λ requires SNR of 10-15dB which is less than that of uniform linear antenna with $d = \lambda/2$, about 25-30dB;

- A large number of snapshot can compensate for a poor SNR. However, a large number of snapshot will require longer processing time;

- With the circular antenna configuration, the MUSIC requires the lowest SNR among the three algorithms as shown in Figures 3a, 3b, 3c.

All the simulations above show that, it is suitable to use the MUSIC algorithm in accordance with a circular phased array antenna in small multi-channel DF devices.

Conclusion

The paper evaluates the performance of small multi-channel DF devices using different phased array antenna configurations and DF algorithms. Simulation results show that the use of MUSIC algorithm in accordance with a circular phased array antenna requires the lowest SNR and the least number of snapshot which can potentially reduce processing time. Therefore, it is recommended that the MUSIC algorithm and circular phased array antenna should be applied in small multi-channel DF devices.

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