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DISCRETE PLAYING OF PERSECUTION WITH LEVEL OF BRIGHTNESS OF DIGITAL IMAGE DESCRIBED BY SECOND ORDER EQUATIONS

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ABSTRACT.

The work is devoted to the study of a class of discrete pursuit games with a digital image level, which is described by systems of second-order equations. Sufficient conditions are obtained for the possibility of completing the pursuit in discrete games with boundary conditions. When solving the problem of pursuit with the level of a digital image, Chebyshev polynomials of the second kind are used.

Keywords: pursuit, pursuit, evader, terminal set, pursuit control, runaway control

1. INTRODUCTION

The approach of applying a two-dimensional second derivative in the tasks of improving the brightness of a digital image is reduced to the choice of a discrete formulation of the second derivative and to the subsequent construction of a filter mask based on this formulation. Isotropic filters are considered, the response of which does not depend on the direction of inhomogeneities in the image being processed. In other words, isotropic filters are invariant to rotate, in the sense that rotating the image and then applying the filter produces the same result as the initial application of the filter and then rotating the result.

The simplest isotropic operator based on derivatives is the Laplacian (the Laplace operator), which in the case of a function of two variables, is defined as

$$\nabla^2 z = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}. \qquad (1)$$

Since derivatives of any order are linear operators, then the Laplacian is also a linear operator.

To apply this equation in digital image processing, it must be expressed in a discrete form. There are several ways to set the Laplacian in discrete form based on the values of the neighboring pixels. The following definition of a discrete second derivative is one of the most commonly used formulas. Taking into account that there are two variables and notation $z(\mathrm{x}, \mathrm{y})\big|_{(\mathrm{x}_i, y_j)} = z\left(\mathrm{x}_i, y_j\right) = z_{i,j}$, for the partial second derivative with respect to x, we get

$$\left.\frac{\partial^2 z}{\partial x^2}\right|_{(\mathrm{x}_i, y_j)} = z\left(\mathrm{x}_{i+1}, y_j\right) - 2z\left(\mathrm{x}_i, y_j\right) + z\left(\mathrm{x}_{i-1}, y_j\right) = z_{i+1,j} - 2z_{i,j} + z_{i-1,j}$$

and, similarly for the partial second derivative y, we get

$$\left.\frac{\partial^2 z}{\partial y^2}\right|_{(\mathrm{x}_i, y_j)} = z\left(\mathrm{x}_i, y_{j+1}\right) - 2z\left(\mathrm{x}_i, y_j\right) + z\left(\mathrm{x}_i, y_{j-1}\right) = z_{i,j+1} - 2z_{i,j} + z_{i,j-1}.$$

The discrete formulation of the two-dimensional Laplacian given by (1) is obtained by combining these two components

$$\nabla^2 z\big|_{(\mathrm{x}_i, y_j)} = -4z\left(\mathrm{x}_i, y_j\right) + \left[z\left(\mathrm{x}_{i+1}, y_j\right) + z\left(\mathrm{x}_{i-1}, y_j\right) + z\left(\mathrm{x}_i, y_{j+1}\right) + z\left(\mathrm{x}_i, y_{j-1}\right)\right] =$$

$$-4z_{i,j} + z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}.$$

So, as the Laplace operator is essentially the second derivative, its use underlines the discontinuity of the brightness levels in the image and suppresses areas with weak changes in brightness. This results in an image containing grayish lines at the site of contours and other gaps superimposed on a dark background without features. But the background can be "restored", while maintaining the effect of sharpening achieved by the Laplacian. To do this, simply add the original image and the Laplacian. It should be remembered which of the definitions of the Laplacian was used. If a definition using negative central coefficients was used, then obtaining the effect of increasing the sharpness, the Laplacian image should be subtracted, not added. Thus, the generalized algorithm of using the Laplacian for image enhancement is as follows.

$$g\left(x_i, y_j\right) = \begin{cases} z\left(x_i, y_j\right) - \nabla^2 z\left(x_i, y_j\right), \text{ если } w(0,0) < 0, \\ z\left(x_i, y_j\right) + \nabla^2 z\left(x_i, y_j\right), \text{ если } w(0,0) \geq 0. \end{cases}$$

Where $g(x, y)$ - processed image, $z(x, y)$ - input image, $w(0,0)$ - value of the central coefficient of the Laplacian mask. Hence it is clear that in order to improve the image as already noted, we must change the ie drive Laplacian

$$\nabla^2 z\left(x_i, y_j\right) = -4z_{i,j} + z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1}.$$

Given the presence of noise, we obtain a model example of discrete games, the process of pursuing the equations described (show. [1] - [3])

$$-4z_{i,j} + z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1} = -u_{i,j} + \upsilon_{i,j}, \left|u_{i,j}\right| \leq \rho, \left|\upsilon_{i,j}\right| \leq \sigma, \sigma < \rho,$$

$$z_{0,j} = 0, \ z_{m+1,j} = 0, \quad z_{i,0} = 0, \ z_{i,\theta} = 0, \quad (*)$$

$$i = 1, 2, ..., m, \ j = 1, 2, ..., \theta - 1,$$

where the left side of the equation is a discrete analog of Laplacian $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$ image brightness functions $z = z(x, y)$, a $z_{i,j} -$ image brightness at a point, (x_i, y_j) т.е. $z_{i,j} -$ the value of the brightness levels of the image of the corresponding pixels (i, j), $u_{i,j}, \upsilon_{i,j} -$ control parameters. Without loss of generality, it is convenient to assume that if either $i = 0,$ either $i = m + 1,$ either $j = 0,$ either $j = \theta,$ that $z_{i,j} = 0,$ those the image is bordered by pixels with zero brightness levels. Harassment is considered complete if $z_{i,j}$ satisfy the condition: $\delta \leq z_{i,j} \leq \delta + \varepsilon, i_0 \leq i \leq i_1, j_0 \leq j \leq j_1$ где $1 \leq i_0, i_1 \leq m, 1 \leq j_0, j_1 \leq \theta - 1$ for some preset $\delta > 0, \varepsilon > 0.$ This means that $z_{i,j}$ the value of the brightness levels of the image in the predetermined pixels was in a certain segment. Using boundary conditions, when $i = 1, 2, ..., m$ from (*) get the system

$$-4z_{1,j} + z_{0,j} + z_{2,j} + z_{1,j-1} + z_{1,j+1} = -u_{1,j} + \upsilon_{1,j},$$

$$-4z_{2,j} + z_{1,j} + z_{3,j} + z_{2,j-1} + z_{2,j+1} = -u_{2,j} + \upsilon_{2,j},$$

$$-4z_{i,j} + z_{i-1,j} + z_{i+1,j} + z_{i,j-1} + z_{i,j+1} = -u_{i,j} + \upsilon_{i,j},$$

$$-4z_{m-1,j} + z_{m-2,j} + z_{m,j} + z_{m-1,j-1} + z_{m-1,j+1} = -u_{m-1,j} + \upsilon_{m-1,j},$$

$$-4z_{m,j} + z_{m-1,j} + z_{m+1,j} + z_{m,j-1} + z_{m,j+1} = -u_{m,j} + \upsilon_{m,j}.$$

Denoting,

$$z_j = (z_{1,j}, z_{2,j}, ..., z_{m,j})^T, u_j = (u_{1,j}, u_{2,j}, ..., u_{m,j})^T, \upsilon_j = (\upsilon_{1,j}, \upsilon_{2,j}, ..., \upsilon_{m,j})^T$$

we have

$$-z_{j-1} + Cz_j - z_{j+1} = u_j - \upsilon_j, 1 \leq j \leq \theta - 1,$$

$$z_0 = 0, \ z_\theta = 0, \quad (**)$$

where $z_j \in R^m$ и $u_j -$ pursuer control parameter; $\upsilon_j -$ escape player control parameter: $u_j \in R^m, \upsilon_j \in R^m$ components that satisfies the condition, $\left|u_{i,j}\right| \leq \rho, \left|\upsilon_{i,j}\right| \leq \sigma, \sigma < \rho,$ and $C -$ Jacobi square - three-diagonal matrix [2]

$$C = \begin{pmatrix} 4 & 1 & & 0 \\ 1 & 4 & 1 & \\ . & . & . & . \\ 0 & & 1 & 4 \end{pmatrix}.$$

$M \subset R^m$ the terminal set that the game ends.

The tasks of pursuit and escape for various classes of differential and discrete games are the subject of numerous studies [4–16]. General questions of the theory of discrete games are considered in monographs [4], [5]. In [6], discrete differential games with information lag were studied, in [7] - [10], the relationship between differential games with distributed parameters and discrete ones was studied. The pursuit and escape problems for linear discrete games were studied in [11] - [14], the escape problem for nonlinear discrete games was studied in [15], [16]. In these papers, the motion of points is described by discrete first-order equations.

The purpose of this work is to study a new class of game problems with discrete second-order equations. For this class of discrete games, sufficient conditions are obtained for the possibility of completing the pursuit when the position of the object is given in the boundary moments. To solve this problem, Chebyshev polynomials of the second kind are used [2], [3].

2. PROBLEM STATEMENT

Instead of the game (**), we will consider a more general discrete game, which is point motion z m dimensional Euclidean space R^m described by equations

$$-z_{j-1} + Cz_j - z_{j+1} = u_j - \upsilon_j, 1 \le j \le \theta - 1, \quad (1)$$

$$z_0 = \phi_0,\ z_\theta = \phi_\theta \quad (2)$$

where j - step number, $C - mxm$ - constant square matrix, u, υ - control parameters, u - chase parameter, υ - escape parameter, $u_j \in P \subset R^m, \upsilon_j \in Q \subset R^m$, P and Q - non-empty sets, parameter u selected as a sequence $u = u(.) = (\mathrm{u}_1, \mathrm{u}_2, ..., \mathrm{u}_{\theta-1}), \mathrm{u}_j \in P, j = 1, 2, ..., \theta - 1,$ parameter υ - as a sequence $\upsilon = \upsilon(.) = (\upsilon_1, \upsilon_2, ..., \upsilon_{\theta-1}), \upsilon_j \in Q, j = 1, 2, ..., \theta - 1.$ In addition, in R^m allocated terminal set M.

Purpose of the pursuing player z_j on the set M, fleeing player tends to put it.

Definition. We will say that in the game (1), (2) from the "boundary" position $(\overline{\varphi}_0, \overline{\varphi}_\theta)$ can complete the persecution for $N \le \theta$ steps, if in any order $\overline{\upsilon}_1, \overline{\upsilon}_2, \ldots, \overline{\upsilon}_{N-1}$ escape control can build such a sequence $\overline{u}_1, \overline{u}_2, \ldots, \overline{u}_{N-1}$ prosecution management, what's the solution $\mathrm{z} = \mathrm{z}(.) = (z_0, \overline{z}_1, \overline{z}_2, \ldots, \overline{z}_{N-1}, z_N)$ the equations

$$-z_{j-1} + Cz_j - z_{j+1} = \overline{u}_j - \overline{\upsilon}_j,\ 1 \le j \le N - 1,$$

$$z_0 = \overline{\varphi}_0,\ z_\theta = \overline{\varphi}_\theta,$$

with some $d \le N$ hits on $M : \overline{z}_d \in M$.

Let a discrete game be described by equations (1), (2). Through - $U_n(x)$ denote the Chebyshev polynomial of the second kind of degree n:

$$U_n(x) = \begin{cases} \dfrac{\sin(n+1)\arccos\ x}{\sin \arccos\ x}, & |x| \le 1 \\ \dfrac{1}{2\sqrt{x^2 - 1}} \left[\left(x + \sqrt{x^2 - 1} \right)^{n+1} - \left(x + \sqrt{x^2 - 1} \right)^{-(n+1)} \right], & |x| > 1. \end{cases} \quad (3)$$

From here you can easily get the following: $U_{-2}(x) = -1$, $U_{-1}(x) = 0$, $U_0(x) = 1$, $U_1(x) = 2x$, etc. In the monographs [2], [3] there are the following recurrence relations

$$U_{n+2}(x) = 2xU_{n+1}(x) - U_n(x),\ n \ge 0,$$
$$U_0(x) = 1,\ U_1(x) = 2x. \quad (4)$$

Further we denote by $U_n(x)$ Chebyshev matrix polynomial from a matrix X, determined by the recurrence formulas (4). From here from (3), (4) we get: $U_{-2}(X)=E$, $U_{-1}(X)=\overline{0}$, $U_0(X)=E$, $U_1(X)=2X$, where E – single, and $\overline{0}$ – zero matrix. Let be $\bar{u}=\bar{u}(j)=\bar{u}_j$, $\bar{\upsilon}=\bar{\upsilon}(j)=\bar{\upsilon}_j$, $1\le j\le N$ – preset controls. If matrix C such that $U_{\theta-1}(\frac{1}{2}C)$ non-degenerate matrix, then solutions of equation (1) with boundary conditions $z_0=\bar{\phi}_0$, $z_\theta=\bar{\phi}_\theta$ determined by the formula (see [2], chap. I, § 4, equation (46))

$$z_n=U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n-1}\left(\frac{1}{2}C\right)\left[z_0+\sum_{k=1}^{n-1}U_{k-1}\left(\frac{1}{2}C\right)\left(u_k-\upsilon_k\right)\right]+$$
$$+U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n-1}\left(\frac{1}{2}C\right)\left[z_\theta+\sum_{k=n}^{\theta-1}U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(u_k-\upsilon_k\right)\right]. \tag{5}$$

3. FORMULATION OF KEY RESULTS

Assumption 1. $M=M_0+M_1$, where M_0 – linear subspace R^m; M_1 – subset of subspace L – orthogonal complement M_0 in R^m. Through Π denote the operation of the orthogonal projection of R^m to L, and through $A+B$ and $A\overset{*}{-}B$ – algebraic sum and geometric difference of sets, respectively [17]. Let be $M_{1,1}+M_{1,2}=M_1$ and

$$W_{1,1}(n)=\sum_{k=1}^{n-1}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)\left(P\overset{*}{-}Q\right)-M_{1,1};$$
$$W_{1,2}(n)=\sum_{k=n}^{\theta-1}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(P\overset{*}{-}Q\right)-M_{1,2}, \tag{6}$$

In here $U_n\left(\frac{1}{2}C\right)$ – Chebyshev matrix polynomial.

Assumption 2. Let there be such $n=n_0\le\theta-1$, that

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right]\in W_{1,1}(n_0)$$

and

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right]\in W_{1,2}(n_0). \tag{7}$$

Theorem 1. If assumptions 1, 2 are fulfilled, then in the game (1), (2) from the "boundary" position (z_0,z_θ) may complete the pursuit of $N(z_0,z_\theta)\le n_0$ steps. Let be $1\le n\le\theta-1$, $W_{2,1}(0)=-M_{1,1}$, $W_{2,2}=-M_{1,2}$,

$$W_{2,1}(n)=\bigcap_{\upsilon(k)\in Q}\left[W_{2,1}(n-1)+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)\left(P-\upsilon(k)\right)\right],$$
$$1\le k\le n-1,$$
$$W_{2,2}(n)=\bigcap_{\upsilon(k)\in Q}\left[W_{2,2}(n-1)+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(P-\upsilon(k)\right)\right],$$
$$n\le k\le\theta-1. \tag{8}$$

Assumption 3. Let there be such $n=n_0\le\theta-1$, that

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right]\in W_{2,1}(n_0)$$

and

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right]\in W_{2,2}(n_0). \tag{9}$$

Theorem 2. If assumptions 3 are fulfilled, then in the game (1), (2) from the "boundary" position (z_0, z_θ) may complete the pursuit of $N(z_0, z_\theta)\le n_0$ steps.

$$\text{Пусть } \alpha_n(\cdot)=\{\alpha_1,\alpha_2,\dots,\alpha_{n-1}:\alpha_k\ge 0,\ \sum_{k=1}^{n-1}\alpha(k)=1\},$$

$$\beta_n(\cdot)=\{\beta_n,\beta_{n+1},\dots,\beta_{\theta-1}:\beta_k\ge 0,\ \sum_{k=n}^{\theta-1}\beta(k)=1\}$$

and

$$W_1(\alpha_n(\cdot))=\sum_{k=1}^{n-1}\left[\left(\alpha_k M_{1,1}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)P\right)\overset{*}{-}\right.$$
$$\left.\overset{*}{-}\Pi U_{\theta-1}\left(\frac{1}{2}C\right)U_{\theta-n-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)Q\right],$$

$$W_2(\beta_n(\cdot))=\sum_{k=n}^{\theta-1}\left[\left(\beta_k M_{1,2}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)P\right)\overset{*}{-}\right.$$
$$\left.\overset{*}{-}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)Q\right].$$

Set

$$W_{3,1}(0)=M_{1,1},\ W_{3,1}(n)=\underset{\alpha_k(\cdot)}{U}W_1(\alpha_k(\cdot)),\ 1\le k\le n-1,$$
$$W_{3,2}(0)=M_{1,2},\ W_{3,2}(n)=\underset{\beta_k(\cdot)}{U}W_1(\beta_k(\cdot)),\ 1\le k\le \theta-1. \tag{10}$$

Assumption 4. Let there be what $n=n_0\le\theta-1$, what

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right]\in W_{3,1}(n_0),$$
$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_0\right]\in W_{3,1}(n_0). \tag{11}$$

Theorem 3. If assumptions 4 are fulfilled, then in the game (1), (2) from the "boundary" position (z_0, z_θ) may complete the pursuit of $N(z_0, z_\theta)\le n_0$ steps.

4. CONCLUSION

Summarizing the obtained results, we conclude that the discrete pursuit game (1), starting from the "boundary" position (2) $z_r|_{r\in\Gamma}=\phi_r$, can be finished for $N(z_0, z_\theta)\le n_0$ steps. Thus, to solve the game problem of pursuit of the form (1), (2), we used the Chebyshev polynomials of the second kind of the form (3) and the recurrent relation (4), the formula (5). The set (6) is a discrete analogue of the so-called first integral of L. S. Pontryagin [17], the inclusion of (7) gives the first sufficient conditions for the possibility of completing the pursuit of the task. The set (8) is a discrete analog of the second integral of L. S. Pontryagin, the inclusion of (9) gives the second sufficient conditions for the possibility of completing the pursuit of the task. Set (10) is a discrete analogue of the third method of N. Yu. Satimov [14], and the inclusion of (11) gives the third sufficient conditions for the possibility of completing the game. In Theorems 1-3, sufficient conditions are obtained for solving the corresponding problems in this form. It should be noted that many problems of mathematical physics can be approximated using problem (1), (2).

ATTACHMENT

Proof of Theorem 1. From (6) and (7) it follows that ***such***

$$a(k) \in \begin{cases} \bigcap\limits_{\upsilon(k)\in Q} \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)\left(P-\upsilon(k)\right), & 1 \le k \le n_0-1, \\ \bigcap\limits_{\upsilon(k)\in Q} \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(P-\upsilon(k)\right), & n_0 \le k \le \theta-1, \end{cases}$$

$$b_1 \in M_{1,1},\ b_2 \in M_{1,2},$$

that

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right] = \sum_{k=1}^{n_0-1} a(k) - b_1,$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right] = \sum_{k=n_0}^{\theta-1} a(k) - b_2. \tag{12}$$

Let be $\upsilon = \bar{\upsilon}(k)$, $1 \le k \le \theta-1$ – arbitrary admissible control of the evader; pursuing player management $u = \bar{u}(k)$ let's build as a solution to the following equation

$$a(k) = \begin{cases} \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)\left(\bar{u}(k)-\bar{\upsilon}(k)\right), \\ \qquad\qquad 1 \le k \le n_0-1; \\ \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(\bar{u}(k)-\bar{\upsilon}(k)\right). \end{cases} \tag{13}$$

It is clear that these equations have optional solutions $a(k)$, because $\bar{\upsilon}(k) \in Q$ and $\bar{u}(k) \in P$. Substituting $\upsilon = \bar{\upsilon}_k = \bar{\upsilon}(k)$ and $u = \bar{u}_k = \bar{u}(k)$ in (1) and applying the formula (5), will get

$$z(n_0) = U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\left[z_0 + \sum_{k=1}^{n_0-1} U_{k-1}\left(\frac{1}{2}C\right)\left(\bar{u}_k-\bar{\upsilon}_k\right)\right] +$$
$$+ U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)\left[z_0 + \sum_{k=n_0}^{\theta-1} U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(\bar{u}_k-\bar{\upsilon}_k\right)\right].$$

Hence, applying to both sides of the equality the design projection operator and from equality (12), (13), we have

$$\Pi z(n_0) = \Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0 + \sum_{k=1}^{n_0-1} U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\cdot\right.$$
$$\left.\cdot U_{k-1}\left(\frac{1}{2}C\right)\left(\bar{u}_k-\bar{\upsilon}_k\right)\right] + \Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta +\right.$$
$$\left.+\sum_{k=n_0}^{\theta-1} U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(\bar{u}_k-\bar{\upsilon}_k\right)\right] =$$
$$= \Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right] + \sum_{k=1}^{n_0-1} a(k) +$$
$$+\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right] + \sum_{k=n_0}^{\theta-1} a(k) = \Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right] -$$
$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right] + b_1 + \Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right] -$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_0\right]+b_2=b_1+b_2\in M_{1,1}+M_{1,2}=M_1\,.$$

From this inclusion we get that $\Pi z(n_0)\in M_1$ and, means, $z(n_0)\in M$. Q.E.D.

Proof of Theorem 2. By the condition of the theorem and from (8), (9) there are such vectors

$$a(k)\in\begin{cases}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\left(P-\upsilon(k)\right), & 1\le k\le n_0-1,\\ \\ \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(P-\upsilon(k)\right), & n_0\le k\le\theta-1,\end{cases}$$

and $b_1\in M_{1,1}$, $b_2\in M_{1,2}$, that

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right]\in W_{2,1}(n)+a(n_0-1)+a(n_0-2)+\ldots+a(n)\,,$$
$$1\le n\le n_0-1,$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right]\in W_{2,2}(n)+a(\theta-1)+a(\theta-2)+\ldots+a(n)\,,n_0\le n\le\theta-1\,.$$

From here we get that

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right]=\sum_{k=1}^{n_0-1}a(k)-b_1\,,$$
$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right]=\sum_{k=n_0}^{\theta-1}a(k)-b_2. \tag{14}$$

Now let $\upsilon=\bar{\upsilon}_k=\bar{\upsilon}(k)$, $1\le k\le\theta-1$ – arbitrary admissible control of the evader; control of the pursuing player $u=\bar{u}_k=\bar{u}(k)$, $\bar{u}(k)\in P$ are constructed as solutions of the following equation

$$a(k)=\begin{cases}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\left(\bar{u}(k)-\bar{\upsilon}(k)\right), 1\le k\le n_0-1,\\ \Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(\bar{u}(k)-\upsilon(k)\right),\ n_0\le k\le\theta-1.\end{cases} \tag{15}$$

Substituting $u=\bar{u}_k=\bar{u}(k)$, $\upsilon=\bar{\upsilon}_k=\bar{\upsilon}(k)$ in (1), (2), considering (14), (15) and simultaneously applying formula (5) and the design operator, we have

$$\Pi z(n_0)=\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0+\sum_{k=1}^{n_0-1}U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\cdot\right.$$
$$\left.\cdot U_{k-1}\left(\frac{1}{2}C\right)\left(\bar{u}_k-\bar{\upsilon}_k\right)\right]+\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta+\right.$$
$$\left.+\sum_{k=n_0}^{\theta-1}U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)\left(\bar{u}_k-\bar{\upsilon}_k\right)\right]=$$
$$=\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right]+\sum_{k=1}^{n_0-1}a(k)+$$
$$+\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_0\right]+\sum_{k=n_0}^{\theta-1}a(k)=b_1+b_2\in M_{1,1}+M_{1,2}=M_1\,.$$

From here we get that $z(n_0)\in M$. Theorem 2 is proved.

Proof of Theorem 3. Instead of inclusion (11), bearing in mind (10), we consider the equivalent inclusion

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right]\in W_1(\bar{\alpha}_{n_0}(\cdot)),$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right]\in W_1(\bar{\beta}_{n_0}(\cdot)),$$

existence $\bar{\alpha}_{n_0}(\cdot)=\{\bar{\alpha}_1,\bar{\alpha}_2,\dots,\bar{\alpha}_{n_0-1}\}$, $\bar{\beta}_{n_0}(\cdot)=\{\bar{\beta}_{n_0},\bar{\beta}_{n_0+1},\dots,\bar{\beta}_{\theta-1}\}$ follows from (11).

From here we get

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right]\in\sum_{k=1}^{n_0-2}\left[\bar{\alpha}_k M_{1,1}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\times\right.$$

$$\left.\times U_{k-1}\left(\frac{1}{2}C\right)P\stackrel{*}{-}\Pi U_{\theta-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)Q\right]+$$

$$+\left(\bar{\alpha}_{n_0-1}M_1+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{n_0-2}\left(\frac{1}{2}C\right)P\right)\stackrel{*}{-}$$

$$\stackrel{*}{-}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\cdot U_{n_0-2}\left(\frac{1}{2}C\right)Q, \tag{16}$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right]\in\sum_{k=n_0+1}^{\theta-1}\left[\bar{\beta}_k M_{1,2}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)\times\right.$$

$$\left.\times U_{\theta-k-1}\left(\frac{1}{2}C\right)P\stackrel{*}{-}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)\cdot U_{\theta-k-1}\left(\frac{1}{2}C\right)Q\right]+$$

$$+\left(\bar{\beta}_{n_0}M_{1,2}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)\cdot U_{\theta-n_0-1}\left(\frac{1}{2}C\right)P\right)\stackrel{*}{-}$$

$$\stackrel{*}{-}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)Q.$$

Now let $\bar{\upsilon}_1,\bar{\upsilon}_2,\dots,\bar{\upsilon}_{\theta-1}$, $\bar{\upsilon}_k\in Q$, $1\le k\le\theta-1$ arbitrary sequence. By virtue of (16) there is such $\bar{\alpha}_{n_0-1}$ and $\bar{\beta}_{n_0}$ that

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_0\right]+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\times$$

$$\times U_{n_0-2}\left(\frac{1}{2}C\right)\bar{\upsilon}_{n_0-1}\in\sum_{k=1}^{n_0-2}\left[\bar{\alpha}_k M_{1,1}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)P\stackrel{*}{-}\right.$$

$$\left.\stackrel{*}{-}\Pi U_{\theta-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\cdot U_{k-1}\left(\frac{1}{2}C\right)Q\right]+$$

$$\left(+\bar{\alpha}_{n_0-1}M_{1,1}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{n_0-2}\left(\frac{1}{2}C\right)P\right), \tag{17}$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_\theta\right]+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\times$$

$$\times\bar{\upsilon}_{n_0}\in\sum_{k=n_0+1}^{\theta-1}\left[\bar{\beta}_k M_{1,2}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)P\stackrel{*}{-}\right.$$

$$\left.\stackrel{*}{-}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)Q\right]+$$

$$+\left(\bar{\beta}_{n_0}M_{1,2}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)\cdot U_{\theta-n_0-1}\left(\frac{1}{2}C\right)P\right).$$

Control $\bar{u}_{n_0-1}\in P$ and $\bar{u}_{n_0}\in P$ let's build as a solution to the following equation

$$\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{n_0-2}\left(\frac{1}{2}C\right)\bar{u}_{n_0-1}+$$

$$+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{n_0-2}\left(\frac{1}{2}C\right)\bar{\upsilon}_{n_0-1}=\bar{\alpha}_{n_0-1}a_1,\ a_1\in M_{1,1},$$

$$\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\bar{u}_{n_0}+$$

$$+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)\bar{\upsilon}_{n_0}=\bar{\beta}_{n_0}b_1,\ b_1\in M_{1,2}.$$

Further, by virtue of (11) and (17) we have

$$-\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_1\Bigg]+\sum_{k=1}^{n_0-2}\Bigg[\bar{\alpha}_k M_{1,1}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)\cdot$$

$$\cdot U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)P\overset{*}{-}\Pi U_{\theta-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)Q\Bigg]+\bar{\alpha}_{n_0-1}a_1,$$

$$-\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_{\theta-1}\Bigg]+\sum_{k=n_0+1}^{\theta-1}\Bigg[\bar{\beta}_k M_{1,2}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)\cdot$$

$$\cdot U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)P\overset{*}{-}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)Q\Bigg]+\bar{\beta}_{n_0}b_1.$$

Similarly, if control $\bar{\upsilon}_{n_0-2}$, $\bar{\upsilon}_{n_0+1}$ becomes known, then the above described method can be constructed to control $\bar{u}_{n_0-2}$, $\bar{u}_{n_0+1}$, providing inclusion

$$-\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)z_2\Bigg]\in\sum_{k=1}^{n_0-3}\Bigg[\bar{\alpha}_k M_{1,1}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)\cdot$$

$$\cdot U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)P\overset{*}{-}\Pi U_{\theta-1}\left(\frac{1}{2}C\right)U_{\theta-n_0-1}\left(\frac{1}{2}C\right)U_{k-1}\left(\frac{1}{2}C\right)Q\Bigg]+$$

$$+\bar{\alpha}_{n_0-1}a_1+\bar{\alpha}_{n_0-2}a_2,\ a_2\in M_{1,1},$$

$$-\Pi\left[U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)z_{\theta-2}\right]\in\sum_{k=n_0+1}^{\theta-1}\Bigg[\bar{\beta}_k M_{1,2}+\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)\cdot$$

$$\cdot U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)P\overset{*}{-}\Pi U_{\theta-1}^{-1}\left(\frac{1}{2}C\right)U_{n_0-1}\left(\frac{1}{2}C\right)U_{\theta-k-1}\left(\frac{1}{2}C\right)Q\Bigg]+$$

$$+\bar{\beta}_{n_0}b_1+\bar{\beta}_{n_0+1}b_2,\ b_2\in M_{1,2},$$

etc. so get

$$\Pi z_{n_0}=\left(\bar{\alpha}_{n_0-1}a_1+\bar{\alpha}_{n_0-2}a_2+\ldots+\bar{\alpha}_1 a_{n_0}\right)+\left(\bar{\beta}_{n_0}b_1+\bar{\beta}_{n_0+1}b_2+\ldots+\bar{\beta}_{n_0+\theta-1}b_{\theta-1}\right)\in$$

$$\in\left(\bar{\alpha}_{n_0-1}+\bar{\alpha}_{n_0-2}+\ldots+\bar{\alpha}_1\right)M_{1,1}+\left(\bar{\beta}_{n_0}+\bar{\beta}_{n_0+1}+\ldots+\bar{\beta}_{n_0+\theta-1}\right)M_{1,2}=M_{1,1}+M_{1,2}=M_1,$$

from here we have

$$z_{n_0}\in M.$$

The theorem is proven completely. **Comment.** If in (*) the game is considered finished when the average value

$$z_{i,j}, \mathrm{i}_0 \le \mathrm{i} \le \mathrm{i}_1, \mathrm{j}_0 \le \mathrm{j} \le \mathrm{j}_1 : z = \frac{1}{\lambda\mu} \sum_{i_0}^{i_1=i_0+\lambda} \sum_{j_0}^{j_1=j_0+\mu} z_{i,j}, 1 \le i_0, i_1 \le m, 1 \le j_0, j_1 \le \theta - 1$$

satisfies the condition $\delta \le z \le \delta + \varepsilon$. Then a corresponding change can easily generalize Theorems 1-3.

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DISCRETE PLAYING OF PERSECUTION WITH LEVEL OF BRIGHTNESS OF DIGITAL IMAGE DESCRIBED BY SECOND ORDER EQUATIONS

Mamatov M.SH.

The work is devoted to the study of a class of discrete pursuit games with a digital image level, which is described by systems of second-order equations. Sufficient conditions are obtained for the possibility of completing the pursuit in discrete games with boundary conditions. When solving the problem of pursuit with the level of a digital image, Chebyshev polynomials of the second kind are used.

Keywords: pursuit, pursuer, evader, terminal set, pursuit control, evasion control

УДК 519.854.3

МЕТОД ПОСТРОЕНИЯ ПРИБЛИЖЁННОГО РЕШЕНИЯ ИНТЕРВАЛЬНОЙ ЗАДАЧИ ЧАСТИЧНО-ЦЕЛОЧИСЛЕННОГО ПРОГРАММИРОВАНИЯ

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